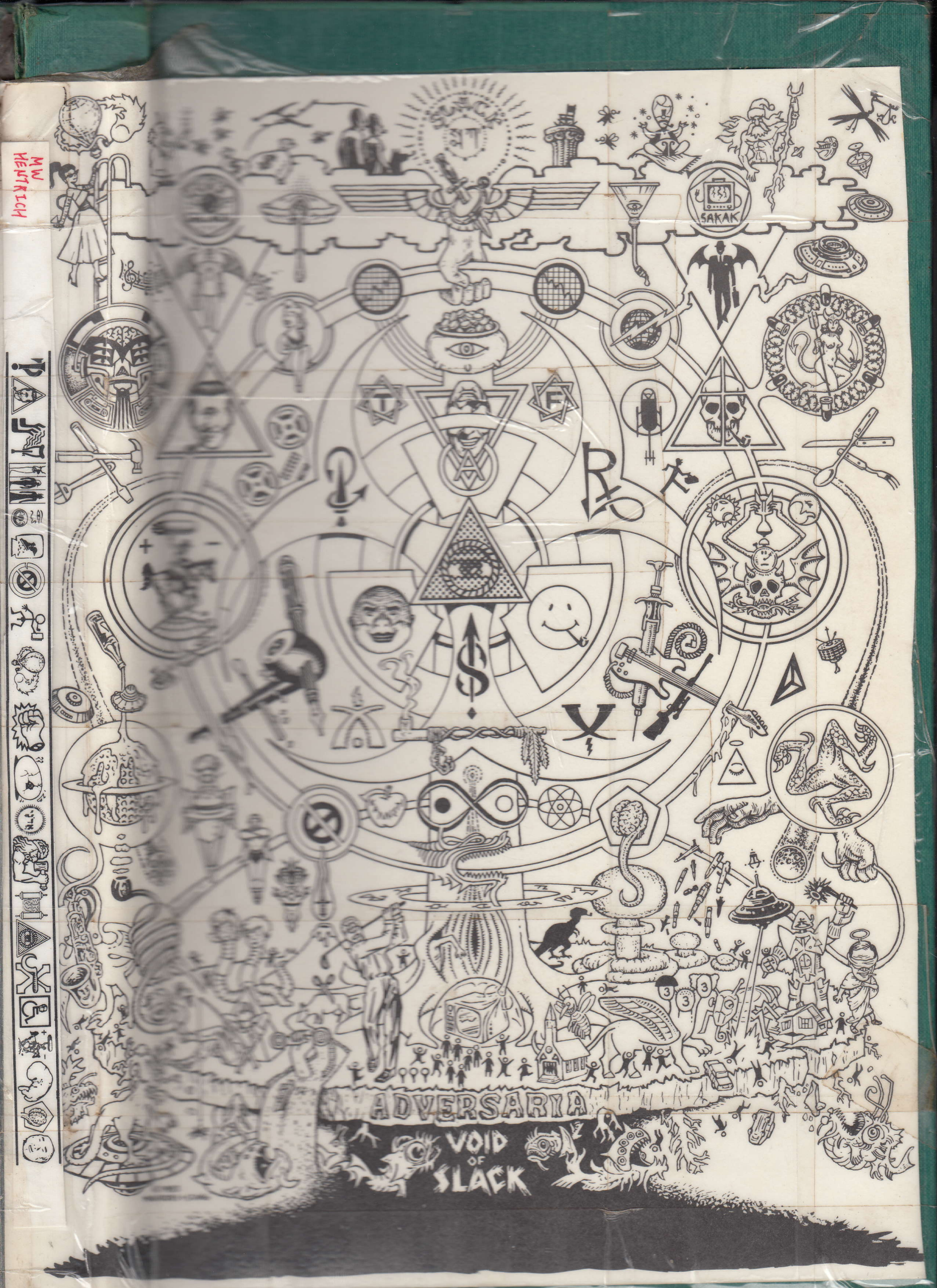
MW
HENRICH
SAKAK
ADVERSARIA
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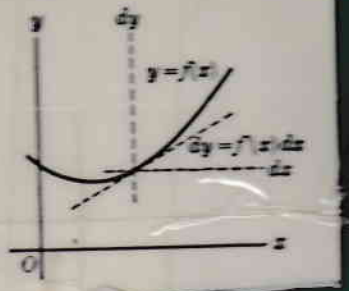
a typical point (x, y) of the curve $y = f(x)$. The axes in this system are scales for the measurement of the variables dx, dy. The equation (1.3–1) has as its graph a straight line of slope $f'(x)$. This line is, of course, the tangent to the curve $y = f(x)$ at the origin of the dx-dy coordinate system.



From (1.3–1) we have the quotient relation

$$\frac{dy}{dx} = f'(x) \tag{1.3–2}$$

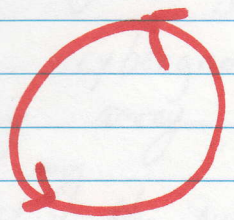
whenever $dx \neq 0$.

EXAMPLE 1. If $y = f(x) = \sin x$, calculate the value of dy

the calculus

L3E = LOGBOOK
OF MY EXPERIENTIAL EXISTENCE
&& EXEGESIS OF VARIOUS TEXTS

COMING AROUND FULL CIRCLE



The symbol above - ⊙ - represents the theme of this volume of L3E - "coming around full circle". It is written in the spirit of the Brainwaves notebooks I kept in 1994 to 1996. I will reenter the calculus I began the introduction back here in December 1998 so as to have a nice jump on the text by the time I fill CYBERGNOSTIC MUTANT... which I forsee as being full by April of 1999.

~~With the Calculus based introductory, this will be filled before Calculus III in September 1999.~~

Even now, I prepare for that semester. It will be what separates me from my fellow programmers. I want to be a MATH WIZARD COMPUTER SCIENTIST - not just a computer geek. What is after "Coming Around Full Circle"? What will the theme of my personal, day to day existence be for my last semester at Brooklake be, when I prepare to take a position in the INDUSTRY? I will be taking PHYSICS II, JAVA, and Operating Systems Technology - a very heavy semester. Shall I wait before coming up with a "theme"? Yes. I will wait until the summer of 1999 before deciding -

Note: No dates will be included in this introduction. Although the "Calculus Review" will extend into the rest of this logbook - and the Calculus Review entries will then include a timestamp, these introductory entries will not be time stamped.

At this present time I am not only preparing for final exams in C, UNIX, and Assembler Language but I am also anxious to begin studying C++. I'm on "at" p 142 in cybergnostic mutant. I will only be pecking away at copying essential material from my "Brainwaves satellites". From now until the summer as well as throughout the summer, besides my formal studies, I will be reviewing my notes from Calculus I (1994) and Calculus II (1995) in preparation for ~~Calculus III~~ PHYSICS II (1999). I will want to be finished with the review by the time ~~Calculus III~~ PHYSICS II begins (September). This is almost a year from now (Nov 27, 1998) but as I want the review to be done in a religious, sacred manner, I will at least start the process now. When I have had enough of writing code and I need a break from computer science, I will turn to my exegesis and prepare for ~~Calculus III~~ Physics II.

Herein - at least in the introduction, there will be no financial status reports ("bitching about how poor I am - I've spent my last $46.00 on this record book") nor philosophical meditations. That begins in section 1 - Now, as Brainwaves_0: MTH171SCRAP is copied into MATHEMATICAL EXERCISES #5, I will be using both MATHEMATICAL EXERCISES #5 & #6 as well as NOTES FROM TEXT MTH-171 and MTH-172. This is no rush.

Note1: Write small if possible. Is it possible?

Note2: Do not neglect C++ or Data Structures or Discrete Mathematics.

$f: \mathbb{D} \to \mathbb{R}$

DOMAIN RANGE

↳ name of function

rule: $f(x) = x^3$

domain: nature of inputs

range: nature of outputs

graphing techniques

get all the information — intercepts (crossing axis)
roots ($f(x) = 0$)
end behavior

$(x-1)^2 (x-3)^5$

zeros are $x = 1$ and $x = 3$
this is an odd function (x^7)
$f(1) = 0$, multiplicity is 2
$f(3) = 0$, multiplicity is 5

at the ends, the graph behaves as its leading coefficient.
at infinity, at both ends, behaves like x^7.

Because x^7 is an odd function, I know that it crosses the graph.

Domain will exclude input that causes division by zero.

$$f(x) = \frac{3x-2}{x^2-3x+2}$$

$\mathbb{D}(f) = \{ x \mid x \text{ is any real number but } x \neq 1, x \neq 2 \}$

as $x \to \infty$, $f(x)$ behaves like $\frac{x}{x^2} = \frac{1}{x}$

Before DERIVE:

$$\frac{x^4-2x+3}{x-2}$$

$$x-2 \overline{)\, x^4+0x^3+0x^2+2x+3}$$

$x^3+2x^2+4x+6+\frac{15}{x-2}$

x^4-2x^3

$2x^3+0x^2$

$2x^3-4x^2$

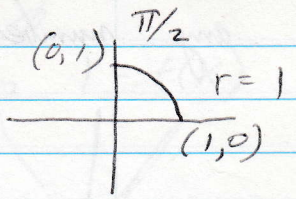
$4x^2-2x$

$4x^2-8x$

$6x+3$

$6x-12$

Trigonometry review: Although $\frac{\pi}{2} = 90°$, $90°$ is not a real number. $\pi/2$ is a real number.

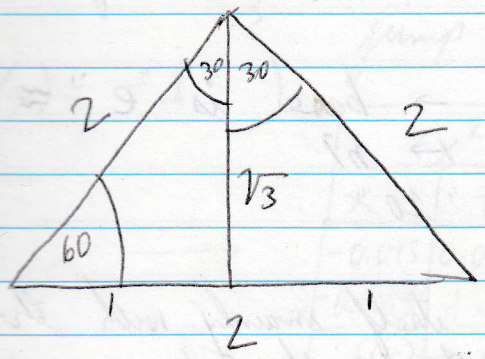


$\sin\theta$ = y coordinate

$\cos\theta$ = x coordinate

$\sin\frac{\pi}{2} = 1$ $\quad \sin\emptyset = \emptyset$ $\quad \tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$

$\cos\frac{\pi}{2} = 0$ $\quad \cos\emptyset = 1$ $\quad \cot\theta = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$

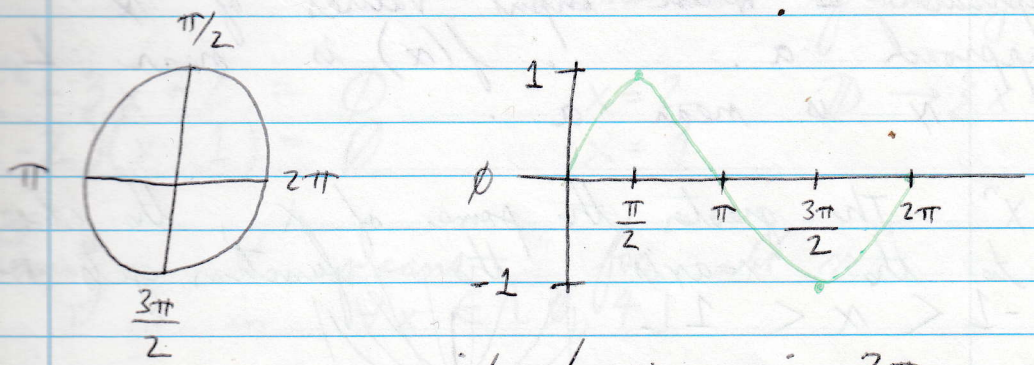


$\sin 60° = \sin\frac{\pi}{3} = \frac{OPP}{HYP} = \sqrt{3}/2$

$\sin 30° = \sin\pi/6 = 1/2$

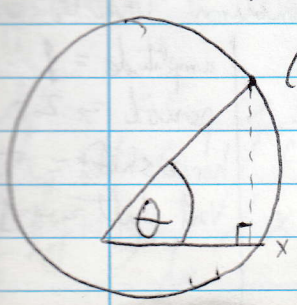
$\cos 60° = \cos\pi/3 = \frac{ADJ}{HYP} = 1/2$

$\cos 30° = \cos\pi/6 = \sqrt{3}/2$

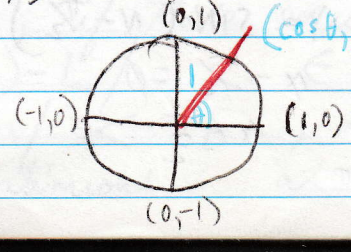


period of $\sin x$ is 2π

a bridge to pass from sine to cosine:



$\sin\theta \leq 1$



$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

LOGARITHMIC FUNCTION

any number to power -1 is reciprocal: $(\frac{1}{2})^{-1} = 2$

1 is itself

$\emptyset$ is 1 : $50^{\circ} = 1$

$2^x = 8 \implies \log_2 8 = x = 3$

The output of the logarithmic function is the EXPONENT.

natural logarithm → base is "e" ≈ 2.718282846...
→ ln

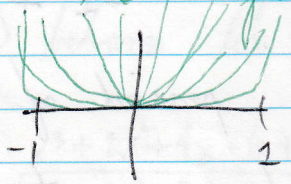
log base is "10".

Calculus concerns itself mainly with the behavior of output values. A mathematician does predictions, observing processes and discovering EVIDENCE graphically, creating proof algebraically.

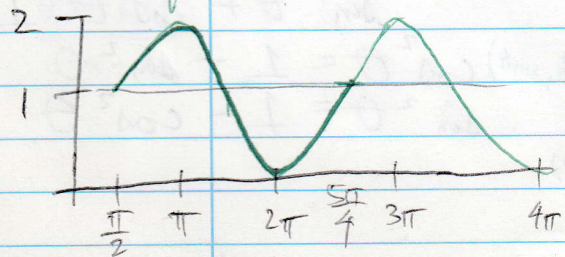
Notation: $\lim_{x \to a} f(x) = L$

Intuitive idea: The output values of a function approach L when input values of x approach a. $f(x)$ is near L when x is near a.

$y = x^n$ The greater the power of x, the closer to the x-axis the function gets when $-1 < x < 1$.



odd function crosses the graph; even function touches graph



$y = \sin(x - \pi/2) + 1$

$x - \pi/2 = \emptyset$

$x = \frac{\pi}{2}$

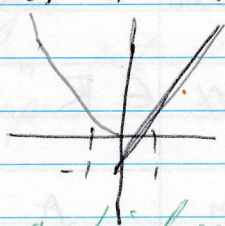
amplitude = 1
period = 2π
horiz shift = $\pi/2$
vert shift = $+1$

Piece wise function $f(x) = \begin{cases} x^2 & x < 0 \\ 2x-1 & x \geq 0 \end{cases}$

analyze local behavior of $f(x)$ as $x \to 0$.

Type into CONVERGE as a function $Y = F(X)$

```
X^2 [FOR X<0]
2X-1 [FOR X>=0]
```



graphical evidence

$\lim_{x \to 0^-} f(x) = 0$

$\lim_{x \to 0^+} f(x) = -1$

} jump discontinuity

numerical evidence →

x^2	
x	$f(x)$
-0.013	0.0001
-0.0003	1×10^{-7}
$f(x) \to 0$	

$2x-1$	
x	$f(x)$
0.013	-0.973
0.004	-0.99
$f(x) \to -1$	

We can analyze a function ALGEBRAICALLY just before we "plug it into" an algebraic mathematical assistant (DERIVE, CONVERGE).

$f(x) = \dfrac{x^2-4}{x^2-3x+2}$

The function will be discontinuous when the denominator equals zero.

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 2$

$x = 1$

$\mathbb{D} \to \{x : x \neq 1, x \neq 2\}$

continuity $\forall$ means "for all x" as in $\forall x \in [0, 4]$

$x \to 4^-$ means "x approaches 4 from the left".

ALGEBRAIC ("BIBLICAL") DEFINITION:

Given $\epsilon > 0$, if there exists a number $\delta > 0$ such that $|f(x) - L| < \epsilon$ for all input values x satisfying $|x - a| < \delta$, then f is said to be continuous at $x = a$.

STOP

SOME BASIC SCIENTIA

Set = a collection of objects

objects = elements, members $x \in A$, $x \notin B$

$x \in \mathbb{R}$ → set of all real numbers

union $A \cup B = \{x: x \in A \text{ OR } x \in B\}$

intersection $A \cap B = \{x: x \in A \text{ AND } x \in B\}$

subset $A \subseteq B$ → every element of A is an element of B.

Interval Notation

$[a, b] = \{x: a \leq x \leq b\}$

$[a, b) = \{x: a \leq x < b\}$

$[a, \infty) = \{x: a \leq x\}$

$(-\infty, b] = \{x: x \leq b\}$

$(a, b) = \{x: a < x < b\}$

$(a, b] = \{x: a < x \leq b\}$.

$(a, \infty) = \{x: a < x\}$

$(-\infty, b) = \{x: x < b\}$

if a and b are endpoints: (a, b) is bounded

(a, ∞) is unbounded

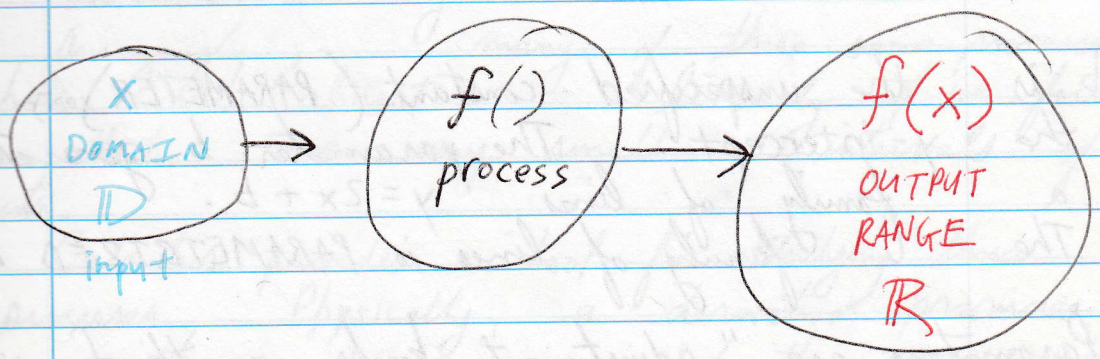
$(-\infty, +\infty)$ has NO ENDPOINTS

Functions and their representations

WISDOM IS A FUNCTION OF EXPERIENCE

a function is a PROCESS that assigns a single output to each member of a given set of inputs. The set of all possible inputs is the DOMAIN ($\mathbb{D}$).

"function as process"



$$f : \mathbb{D} \longrightarrow \mathbb{R}$$

$$x \longmapsto f(x)$$

"x is assigned to f of x"

example AREA_OF_CIRCLE $= \pi$ RADIUS^2

AREA_OF_CIRCLE : $[0, \infty) \longrightarrow \mathbb{R}$

RADIUS $\longmapsto$ AREA_OF_CIRCLE (RADIUS)

as in $x \longmapsto f(x)$

if AREA_OF_CIRCLE $= A$ and RADIUS $= r$

$A(r) = \pi r^2$

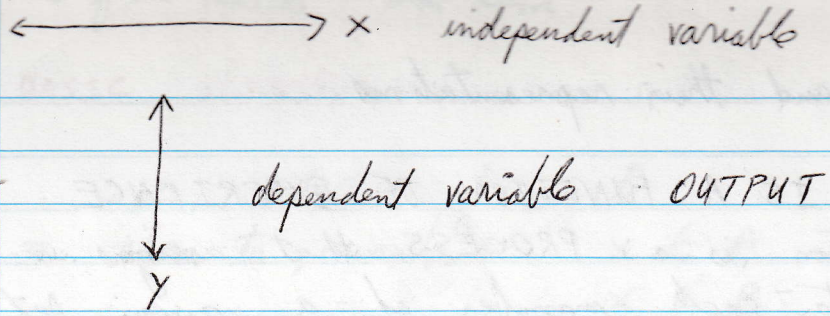
example of notation: (a, b) ordered pair of inputs and outputs

$f = \{ (x, f(x)) : x \in \mathbb{D} \}$

let $y = f(x)$ for each $x \in \mathbb{D}$

x is the independent variable (argument)

y is the dependent variable

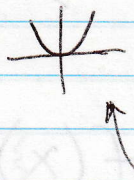


parameter - a constant that varies

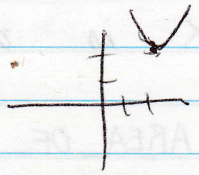
$f : \mathbb{R} \to \mathbb{R}$ where $f(x) = 2x + b$

b is the unspecified constant (PARAMETER), the y-intercept. The parameter b generates a family of lines $y = 2x + b$.
The family of lines is PARAMETRIZED by b.

Parameters are "adjustment knobs" on the function process.

example: $f(x) = x^2$ → 

$f(x) = (x+a)^2 + b$
when $a = 0$ and $b = 0$, $y = x^2$.

when $a = -2$ and $b = 1$ 

changing a moves graph left (+) or right (−).

changing b moves graph up (+) or down (−).

f is even if outputs $f(x) = f(-x)$ x^2

f is odd if outputs $f(x) = -f(-x)$ x^3

The graph of an odd function has symmetry with respect to the origin.

The graph of an even function will have a graph that is symmetric about the y-axis.

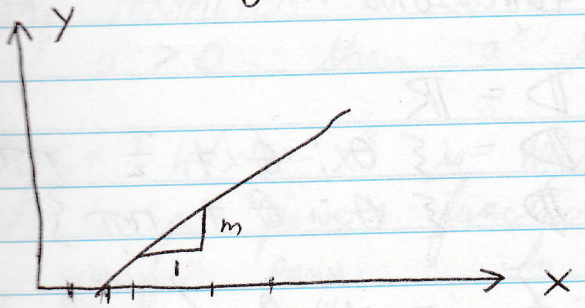
Linear and Locally Linear Functions:

A linear function has a straight line $y = mx + b$ for a graph; and the slope m of that line tells us the rate of change of the function — that is, HOW FAST (and in what direction) the y values (outputs) change as the x values (inputs) increase.

There are real world processes that are simply not described well by linear models.

Amazingly, many of these same processes behave "almost" linearly if we focus our attention over very small intervals of inputs.

Calculus provides a tool for studying these processes. Physically, a derivative provides us with a measure of the INSTANTANEOUS RATE OF CHANGE of a function's outputs; geometrically, it provides the slope of the function's graph at each point.



1 unit change in x results in m units change in y

"two-point formula" $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

"slope-intercept form" $y = mx + b$

horizontal lines: $m = 0$

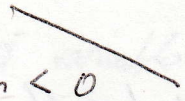
vertical lines: no defined slope

parallel lines: $m_1 = m_2$

perpendicular lines? $m_1 = -\dfrac{1}{m_2}$

$m_1 \cdot m_2 = -1$

$m > 0$
y increases
as x increases



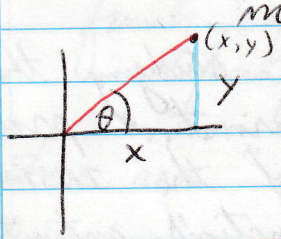
$m < 0$
y decreases
as x increases

point-slope form: $y - y_0 = m(x - x_0)$
we use ~~this~~ in Physics $y - y_0 = \blacktriangledown(x - x_0)$

Taylor form: $f(x) = f(x_0) + m(x - x_0)$

We use trigonometry to relate the slope m of the graph of a line $y = mx + b$ to its angle of inclination θ "theta"

$m = \tan\theta$



$\sin\theta = y$ coordinate
$\cos\theta = x$ coordinate
$\tan\theta = y/x$

DOMAINS OF TRIG FUNCTIONS:

sin and cos: $\mathbb{D} = \mathbb{R}$
tan and sec: $\mathbb{D} = \{\theta : \theta \neq \frac{\pi}{2} + n\pi\}$
cot and csc: $\mathbb{D} = \{\theta : \theta \neq n\pi\}$

$\tan\theta$, by definition, $= \sin\theta / \cos\theta$ for all values $\theta \neq \frac{\pi}{2} + n\pi$

$\sin^2\theta + \cos^2\theta = 1$ because $x^2 + x^2 = 1$ for any point (x, y) on the unit circle.

PYTHAGOREAN IDENTITIES: $\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2(\theta) = \csc^2\theta$

Fundamental Identities:

$\tan\theta = \sin\theta / \cos\theta$ $\qquad \sec\theta = 1/\cos\theta$

$\cot\theta = \cos\theta / \sin\theta$ $\qquad \csc\theta = 1/\sin\theta$

Even / Odd Identities:

$\sin(-\theta) = -\sin(\theta)$

$\cos(-\theta) = \cos(\theta)$

Double Angle Identities:

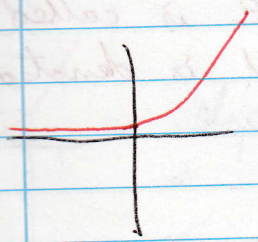
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$

Exponential Functions:

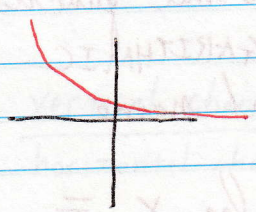
$f: \mathbb{R} \to \mathbb{R}$, $f(x) = a^x$, $(a > 0,\ a \neq 1)$
the PARAMETER a is called the BASE.
if $a > 0$, then a^x is positive for all x

Note: $h(x) = x^3$ is NOT an exponential function. It is a powers function. (input x raised to a specific power. In an exponential function, the input x IS THE POWER.

The graph of $y = a^x$ has one of two shapes depending on whether $a < 1$ or $a > 1$.



$y = 2^x$



$y = \left(\frac{2}{3}\right)^x$

The natural exponential function

$$\exp(x) = e^x$$

irrational number (euler) base $e = 2.71828\ldots$

The natural exponential function exp is the most important function in calculus.

Logarithmic Functions

$$f: \{x: x > 0\} \longrightarrow \mathbb{R}$$
$$f(x) = \log_a x \quad (a > 0,\ a \neq 1)$$

PARAMETER a is the base of the logarithm,
DOMAIN is all positive real numbers.

$$\mathbb{D} = \{x: x > 0\}$$

a logarithmic function is equal to an exponential equation.

$$y = \log_a x \text{ if and only if } x = a^y$$

$\log_3 9$ ⊛ TRANSLATE EQUATION $b = \log_a c$ to $a^b = c$

if $b = \log_3 9$, then $3^b = 9 \therefore b = 2$

$\log_8 2$ ⊛ TRANSLATE $b = \log_a c$ to ⊛ $a^b = c$

if $b = \log_8 2$, then $8^b = 2 \therefore b = \frac{1}{3}$

The logarithmic function with base e is called the NATURAL LOGARITHMIC FUNCTION and is denoted by the special name $\ln$

$$\ln x = \log_e x$$

The natural logarithm and exponential functions "undo" each other.

$\exp(\ln x) = x \qquad \ln(\exp(x)) = x$

The COMMON LOGARITHMIC FUNCTION is $\log_{10}$

$$\log_3 19 = \frac{\log_{10} 19}{\log_{10} 3} = 2.68014\ldots$$

think $b = \log_a c \longrightarrow a^b = c$

$10^b = 19$ $\qquad$ $10^b = 3$

An example of ANALYZING A FUNCTION

$$f(x) = \frac{x^2 - 4}{x^2 - 3x + 2}$$

Analyze ALGEBRAICALLY first! Don't just plug it into a computer.

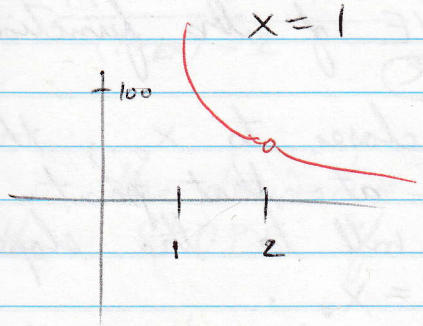
This function will be discontinuous when the denominator equals zero.

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 2$

$x = 1$

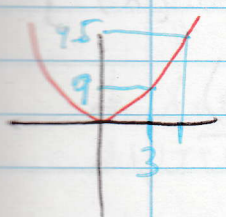


Always try to reason algebraically first before seeking graphical representation.

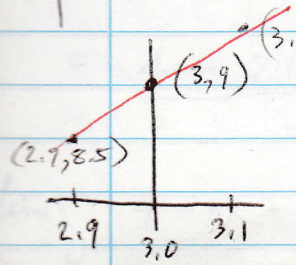
$f(x) = x^2$ $\qquad$ Show that $\lim_{x \to 3} x^2 = 9$

pick ε to be 0.5 $\qquad$ find δ

ε epsilon
δ delta



slope formula: $\dfrac{\text{vertical change}}{\text{horizontal change}}$

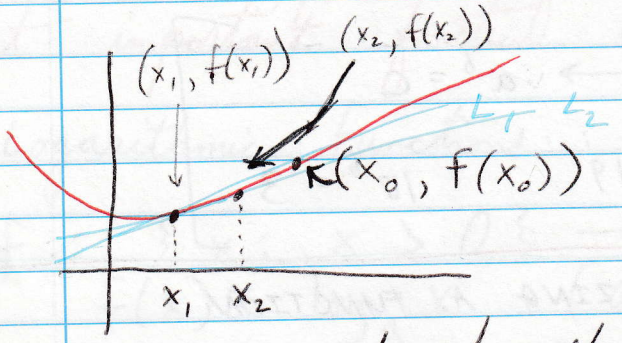


$$m = \frac{9.62 - 9.0}{3.1 - 3.0} = \frac{0.62}{0.1} = 6.2$$

Choose $\delta < \dfrac{\varepsilon}{|m|} = \dfrac{0.5}{6.2} = 0.0806$

For any $\varepsilon > 0$, if there exists a $\delta > 0$ such that $|f(x) - f(a)| < \varepsilon$ whenever $|x - a| < \delta$, then we say that the $\lim_{x \to a} f(x) = f(a)$ and f is continuous at $x = a$.

THE DERIVATIVE



L_1 has slope m_1

$$m_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

L_2 has slope m_2

$$m_2 = \frac{f(x_2) - f(x_0)}{x_2 - x_0}$$

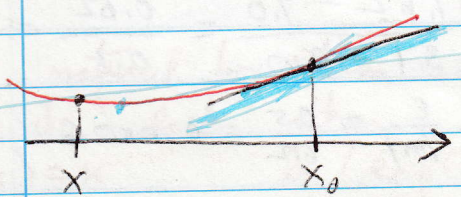
We can continue like this and choose x_3, x_4, etc and find corresponding slopes. If we look at these slopes and find a limit as we pick x_1, x_2, x_3, etc. closer and closer to x_0, then the LIMIT is called the DERIVATIVE of the function $f(x)$. (X)

As the x values get closer to x_0, the limit becomes a tangent line at that point. The limit of the slopes will be the slope of the tangent line at $x = x_0$.

DEFINITION: Let $f(x)$ be a function, then if $\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$ exists, then we say that the derivative exists [the limit is written as $f'(x)$]

and we write, $f'(x) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$

as $x \to x_0$, line becomes tangent.



Numerical Considerations

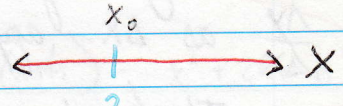
$$\lim_{x\to 2}\frac{x^2-4}{x-2}=\lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}=\lim_{x\to 2}x+2=4$$

$f'(2)=4$

x	$\frac{f(x)-f(2)}{x-2}$	
$x\to 2^-$ approaching from left	1.8	3.8
	1.9	3.9
	1.95	3.95
	1.99	3.99
$x\to 2^+$ approaching from right	2.01	4.01
	2.03	4.03
	2.04	4.04

approaches 4

Other Variation of the Definition:



Δ is upper case δ (change) (change in "x" $\to x_n - x_0$)

$$\lim_{\Delta x\to 0}\frac{f(2+\Delta x)-f(2)}{(2+\Delta x)-2}=\lim_{\Delta x\to 0}\frac{f(2+\Delta x)-f(2)}{\Delta x}=f'(2)$$

$f'(2)$ is the derivative of $f(x)$ at $x=2$, the SLOPE of $f(x)$ at $x=2$.

Generally, the LIMIT of the slopes is the derivative (slope) of $f(x_0)$ at x_0.

PROOF: $f(x)=x^2$ at $x=2$

first look at the quotient: $f(2+\Delta x)\stackrel{?}{=}(2+\Delta x)^2$

$f(2)\stackrel{?}{=}4$

Go back to the quotient and substitute new outputs.

QUOTIENT IS $\frac{f(2+\Delta x)-f(2)}{\Delta x}=\frac{(2+\Delta x)^2-4}{\Delta x}$

$$=\frac{4+2(2)\Delta x+\Delta x^2-4}{\Delta x}=\frac{4\Delta x+\Delta x^2}{\Delta x}=\boxed{4+\Delta x}$$

simplified quotient

$$\lim_{\Delta x\to 0}4+\Delta x=4$$

There are many applications ... see Mathematical Exercises #5 and NOTES FROM TEXT composition books 171 & 172. Physics uses calculus. In fact, the problems of physics caused the need for calculus.

Real world problems are not linear $y = mx + b$, so we focus on very small intervals of inputs that are "almost" linear in their behavior (output).

Leibniz notation for the derivative: if $y = f(x)$, then we write $\frac{dy}{dx} = f'(x) = y'$

definition of the derivative using Leibniz notation:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

The single quantity of $\frac{dy}{dx}$ is the limit of the ratios $\frac{\Delta y}{\Delta x}$ as Δx approaches zero.

Think of an operator acting on the function represented by y:

$$\frac{dy}{dx} = \frac{d}{dx}(y) = y'(x) \quad \text{where } \frac{d}{dx}$$

is read as "the derivative with respect to x" and operates on the function expressed by y to produce the derivative of y'.

also, $dy = y'(x)\,dx$

Written as dy and dx, dy and dx are called differentials. Historically, early users of calculus thought of dy and dx as infinitesimally small increments in input (dx) and output (dy).

The benefit of prime notation $f'(x_0)$ is that it specifies a specific point. To remedy this Leibniz notation for the derivative at $x = x_0$:

$$\left.\frac{dy}{dx}\right|_{x=x_0} = y'(x_0)$$

dependent variable y
derivative operator d/dx
specific point x_0

NOTE: I will be going through some Calculus software tests and doing the work so as to slowly prepare for Calculus in September 1999.

$y = x^5$

$y' = 5x^4$

prove it

$f(x) = x^5 \qquad f'(x) = \dfrac{(x+h)^5 - x^5}{h}$

$(x+h)(x+h) = x^2 + 2hx + h^2 \longrightarrow (x+h)^2$

$(x^2 + 2hx + h^2)(x^2 + 2hx + h^2) = x^4 + 2hx^3 + x^2h^2$

$+ 2hx^3 + 4h^2x^2 + 2h^3x$

$+ h^2x^2 + 2h^3x + h^4$

$= x^4 + 4hx^3 + 6x^2h^2 + 4h^3x + h^4 \longrightarrow (x+h)^4$

$(x+h)(x^4 + 4hx^3 + 6x^2h^2 + 4h^3x + h^4) \longrightarrow (x+h)^5$

$= x^5 + 4hx^4 + 6x^3h^2 + 4h^3x + h^4x$

$+ hx^4 + 4h^2x^3 + 6x^2h^3 + 4h^4x + h^5$

$= x^5 + 4x^4 + 6x^3h^2 + 4h^3x + h^4x + x^4h + 10x^3h^2$

$+ 6x^2h^3 + 4h^4x + h^5$

subtract x^5 from this and divide by h

$4x^4 + 6x^3h + 4h^2x + h^3x + x^4 + 10x^3h$

$+ 6x^2h^2 + 4h^3 + h^4$

as h approaches 0 $\left(\lim_{h \to 0} \dfrac{f(x+h) - f(x_0)}{h}\right)$

$\lim_{h \to 0} \dfrac{(x+h)^5 - x^5}{h} = \boxed{5x^4}$

$\lim_{x \to 0}$

I will show proofs (work) when doing exercises, but for now I will use $f'(x) = nx^{n-1}$.

$A = \pi r^2$ find $A'(r) = \frac{dA}{dr}$

The derivative of Area of a Circle with respect to its radius: $A'(r) = \lim_{\Delta r \to 0} \frac{\Delta A}{\Delta r} = \lim_{\Delta r \to 0} \frac{A(r+\Delta r) - A(r)}{\Delta r}$

$$\lim_{\Delta r \to 0} \frac{\pi (r + \Delta r)^2 - \pi r^2}{\Delta r}$$

$$= \lim_{\Delta r \to 0} \frac{\pi r^2 + 2\pi r \Delta r + \pi \Delta r^2 - \pi r^2}{\Delta r}$$

$$= \lim_{\Delta r \to 0} \frac{2\pi r \Delta r + \pi \Delta r^2}{\Delta r}$$

$$= \lim_{\Delta r \to 0} 2\pi r + \pi \Delta r = 2\pi r$$

Area is a function/process and the circumference of a circle is the derivative of its area, whereas the Volume of a sphere is a function of another dimension of the Calculus.

Before further review of Notes, go through StudyWare:

① What are all the points where $1/(2 - \sin x)$ is discontinuous?

answer: when $2 - \sin x = 0$

∴ when $\sin x = 2$ which is NEVER

There are no points where it is discontinuous.

② What is the limit of $(1/n - 1/n^2)$ $0, \frac{1}{2} - \frac{1}{4}, \frac{1}{3} - \frac{1}{9}, \frac{1}{4} - \frac{1}{16}$? $\boxed{0}$

③ $$\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2 = \boxed{3x^2}$$

(4) $\lim_{h\to 0} \frac{2(x+h)^2 - 2x^2}{h} = \lim_{h\to 0} \frac{2(x^2+2xh+h^2) - 2x^2}{h}$

$\lim_{h\to 0} = \frac{2x^2+4xh+2h^2-2x^2}{h} = \lim_{h\to 0} 4x + 2h = 4x$

The easier method is $\frac{d}{dx} x^n = n x^{n-1}$

$$\boxed{\frac{d}{dx} 2x^2 = 4x}$$

(5) $\lim_{x\to 2} \frac{(x^2+x-6)}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+3)}{x-2}$

$= \lim_{x\to 2} x+3 = 5$

(6) $\lim_{x\to 1} \frac{(\sqrt{x}-1)}{x-1} = \lim_{x\to 1} \frac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)} =$

$\lim_{x\to 1} \frac{1}{\sqrt{x}+1} = \frac{1}{2}$

(7) $\lim_{x\to 6^+} \frac{x+3}{x^2-3x-18} = \lim_{x\to 6^+} \frac{x+3}{(x+3)(x-6)} = \infty$

(8) $\frac{d}{dx} \frac{x^3}{3} = \frac{d}{dx} \frac{1}{3}x^3 = x^2$

(9) $\frac{d}{dx}\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = \frac{d}{dx} x^{\frac{1}{2}} - x^{-\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-}$

$= \frac{1}{2x^{\frac{1}{2}}} + \frac{1}{2x^{3/2}}$ $\quad (x^b)' = bx^{b-1}$

$= \left(1 + 1/\sqrt{x}\right) / 2x$

(7) $\frac{d}{dx}(x^3-1) \stackrel{?}{=} \frac{d}{dx}(3x^2-3x+1)$

$3x^2 = 6x-3 \Rightarrow 3x^2-6x+3=0$

$3(x^2-2x+1)=0 \Rightarrow 3(x-1)(x-1)=0 \Rightarrow x=1$

(11) $\frac{d}{dx}|x| = \frac{d}{dx}x$ and $\frac{d}{dx}-x$

$\therefore \frac{d}{dx}|x| = \frac{x}{|x|}$

(12) $\frac{d}{dx}(5x^2-3x)/(5x-3) = \frac{d}{dx}\frac{x(5x-3)}{(5x-3)} = 1$

(13) $\frac{d}{dx}(x+1)^2-(x-1)^2 = \frac{d}{dx}(x^2+2x+1)-(x^2-2x+1)$

$= \frac{d}{dx}4x = 4$

$\left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}$

(14) $\frac{d}{dx}\left(7x^4-21x^2\right)/\left(3-x^2\right)$

$\frac{d}{dx}\frac{-7x^2(-x^2+3)}{(-x^2+3)} = \frac{d}{dx}-7x^2 = \boxed{-14x}$

(15) $(\sin x)''''$

$(\sin x)' = \cos x \qquad (\cos x)' = -\sin x$

$\therefore (\sin x)^4 = \sin x$

NEXT: PRODUCT & QUOTIENT RULES

note: I will be taking PHYSICS II (not CALCULUS III) in September

Paradox and Contradictions

So far in my study of computer science, the guts of programming are mathematical processes — and the power of the implementation of these processes are in logical, step by step recipes, algorithms; and yet the human brain uses a web of interconnected processing elements called neurons to process information. Each neuron is autonomous and independent. It does its work asynchronously, that is, without any synchronization to others events taking place.

The solution to the problem of recognizing a face in a crowd is NONALGORITHMIC, that is, you cannot devise a step-by-step algorithm to give you the answer; and the data provided to the problems is complex and may be noisy or incomplete.

The vast processing power inherent in biological neural structures has inspired the study of the structure itself for hints on organizing man-made computing structures.

Artificial neural networks covers the way to organize synthetic neurons to solve the same kind of difficult, non-algorithmic problems in the same manner as we think the human brain may. Meanwhile, I am studying how to design algorithmic solutions to problems; and I will be studying Discrete Mathematics over the summer starting June 7 (or 14). There is this paradox forming. Neural Networks require fuzzy logic, and I will be studying Boolean logic. It is important to study both.

Wouldn't a scientist need to understand algorithmic methods and Boolean (binary, digital) logic so as to be fully cognizant of its inability to deal with noise and "degrees of membership between 0 and 1".

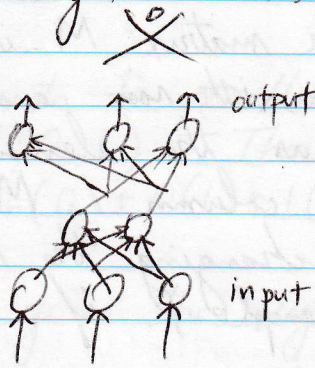
Choosing 0 or 1 is powerful for algorithmic designs; but for nonalgorithmic designs dealing with analog data, we assign degrees of membership between 0 and 1.

I am Noumenon, and I am Abraxas coming around full circle. Keeping the "abraxian nature" of reality in mind, let us not favor analog and fuzzy OVER digital and discrete, but rather, let us recognize the power of both $\{0, 1\}$ and $[0, 1]$.

DIGITAL $\{0, 1\}$ set: 0 (OR and) 1
BINARY, DISCRETE, BOOLEAN

ANALOG $[0, 1]$ interval: between 0 and 1.
FUZZY, DEGREES OF MEMBERSHIP

Proceed with delight rather than frustration.



Patterns can consist of binary digits in the discrete cases, or real numbers representing analog signals in continuous cases. A neural network is a group of processing elements where one subgroup makes independent computations and passes the results to a second subgroup. Each subgroup may in turn make its independent computations and pass on the results to yet another subgroup. Finally, a subgroup of one or more processing elements determines the output from the network. Each processing element makes its computation based upon a weighted sum of its inputs.

Processing in neural networks is done in parallel, rather than sequentially, as would be necessary in computing on digital computers. Computing on digital computers also requires precise information and ALGORITHMS. These observations show the greater potential of neural networks in problem solving. A threshold function determines the output of a neuron. If it is 1, it is said to fire. If 0, it is said to have not fired.

Mathematical Background

Dot Product or Scalar Product of Two Vectors

given vectors U and V, where $U = (u_1, \ldots, u_n)$ and $V = (v_1, \ldots, v_n)$, then dot product or scaler product is $U \cdot V = u_1 v_1 + \ldots + u_n v_n$

A matrix with m rows and n columns is an $m \times n$ matrix. The element in the ith row and jth column of the matrix is referred to as the ij element of the matrix and is typically denoted by a_{ij}.

The transpose of a matrix M is denoted by M^T. The element in the ith row and jth column of M^T is the same as the element of M in its jth row and ith column. M^T is obtained from M by interchanging the rows and columns of M. For example, if

$$M = \begin{matrix} 2 & 7 & -3 \\ 4 & 0 & 9 \end{matrix}, \text{ then } M^T = \begin{matrix} 2 & 4 \\ 7 & 0 \\ -3 & 9 \end{matrix}$$

The addition of matrices is possible only if they have the same dimensions.

$$\begin{matrix} 3 & -4 & 5 \\ 2 & 3 & 7 \end{matrix} + \begin{matrix} 5 & 2 & -3 \\ 6 & 0 & 4 \end{matrix} = \begin{matrix} 8 & -2 & 2 \\ 8 & 3 & 11 \end{matrix}$$

For multiplication $A * B \rightarrow AB$, the number of columns in A has to be equal to the number of rows in B.

$$\text{let } A = \begin{matrix} 3 & -4 & 5 \\ 2 & 3 & 7 \end{matrix} \qquad \text{let } B = \begin{matrix} 5 & 6 \\ 2 & 0 \\ -3 & 4 \end{matrix}$$

$$AB = \begin{matrix} -8 & 38 \\ -5 & 40 \end{matrix} \quad \text{and} \quad BA = \begin{matrix} 27 & -2 & 67 \\ 6 & -8 & 10 \\ -1 & 24 & 13 \end{matrix}$$

Neural Network Construction:

There are three aspects to the construction of a neural network. Structure (how many layers and what their functions are), Encoding (the paradigm used for determining weights on the connections between neurons), Recall (getting an expected output from a given input).

a Hopfield Network

example: single layer of 4 neurons
2 input patterns – binary {0, 1}

note: 2 vectors are said to be orthogonal if their dot product is 0.

The two patterns to be recalled: $A = (1, 0, 1, 0)$ and $B = (0, 1, 0, 1)$. These are orthogonal as $A \cdot B = 0(1) + 1(0) + 0(1) + 1(0) = 0$.

The matrix W gives the weights on the connections in the network:

$$W = \begin{matrix} 0 & -3 & 3 & -3 \\ -3 & 0 & -3 & 3 \\ 3 & -3 & 0 & -3 \\ -3 & 3 & 3 & 0 \end{matrix}$$

The threshold function: $f(t) = \begin{cases} 1 \text{ if } t >= \theta \\ 0 \text{ if } t < \theta \end{cases}$ where $\theta = 0$

$\theta = 0$ is the "threshold value"
It could be any value – here it happens to be zero.

if we present $A = (1, 0, 1, 0)$ as input, the activation at the first node is the dot product of the input vector and the first column of the weight matrix.

first node $\longrightarrow 0 \cdot 1 + (-3 \cdot 0) + (3 \cdot 1) + (-3 \cdot 0) = 3$
second node $\longrightarrow (-3 \cdot 1) + (0 \cdot 0) + (-3 \cdot 1) + (3 \cdot 0) = -6$
third node $\longrightarrow (3 \cdot 1) + (-3 \cdot 0) + (0 \cdot 1) + (-3 \cdot 0) = 3$
fourth node $\longrightarrow (-3 \cdot 1) + (3 \cdot 0) + (-3 \cdot 1) + (0 \cdot 0) = -6$

$f(3) = 1$, $f(-6) = 0$, $f(3) = 1$, $f(-6) = 0$

the output is the same as the input pattern.
When B is input, B is recalled.

The distance formula for the distance between two ~~points~~ two-dimensional points is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.
The distance formula for the distance between two four-dimensional points is therefore:

$$\sqrt{(w_2-w_1)^2+(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

Hence,
If (a,b,c,d) and (e,f,g,h) are two four-dimensional points, the distance between them is the square root of
$(a-e)^2+(b-f)^2+(c-g)^2+(d-h)^2$

if $C=(0,1,0,0)$, the network will recall B because C is closer to B than to A.
$B=(0,1,0,1)$ $A=(1,0,1,0)$.

distance between A and C $\rightarrow \sqrt{1^2+(-1)^2+1^2+0^2}=\sqrt{3}$
distance between B and C $\rightarrow \sqrt{0^2+0^2+0^2+1^2}=\sqrt{1}$

The network handles "noise".

Above we considered the "Euclidean distance" when talking about the closeness of one bit pattern to another bit pattern. Euclidean distance need not be considered. Instead, the HAMMING DISTANCE can be used, which is much easier to determine, since it is the number of bit positions in which the two patterns being ~~computed~~ compared differ. Patterns being strings, Hamming distance is more appropriate than the Euclidean distance.

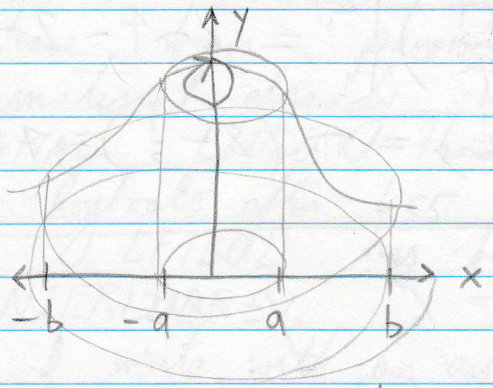
Suppose we want patterns $E=(1,0,0,1)$ and $F=(0,1,1,0)$ also recalled. We need to train the network, and we need to come up with a learning algorithm.
Yes, I said ALGORITHM. We can bring algorithmic logic with us. It will prove to be useful.

Calculating volume using cylindrical shells

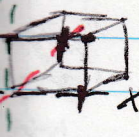
note: In method of disks, slices of the region are $\perp$ to the axis of rotation. We may want to set up the volume using SLICES $\parallel$ (parallel) to the axis of rotation.

Ease of integration of resulting volume integral
OR
Easier to set up initially

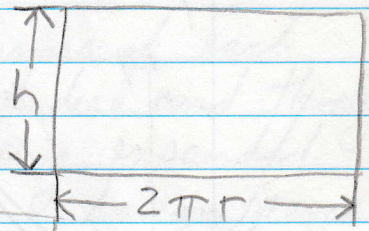
if so, we use a method called "SHELLS"



cylindrical shell



$\therefore$ Area of cylinder $= 2\pi r h$



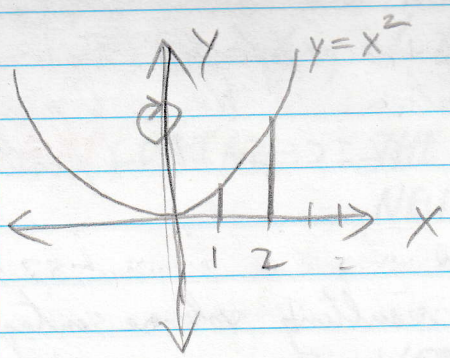
$$\text{Volume} = V = \int_a^b 2\pi r h \, \Delta x$$

$$V = \int_a^b 2\pi \underbrace{r(x)}_{\text{VARIABLE DISTANCE FROM AXIS OF ROTATION}} \underbrace{h(x)}_{\text{HEIGHT OF STRIP} = f(x)} \underbrace{dx}_{\text{THICKNESS}}$$

$$V = 2\pi \int_a^b r(x) f(x) \, dx$$

where a is the location of the innermost shell and b is the location of the outermost shell.

rotate $y = x^2$, $x = 1$, $x = 2$ about y-axis



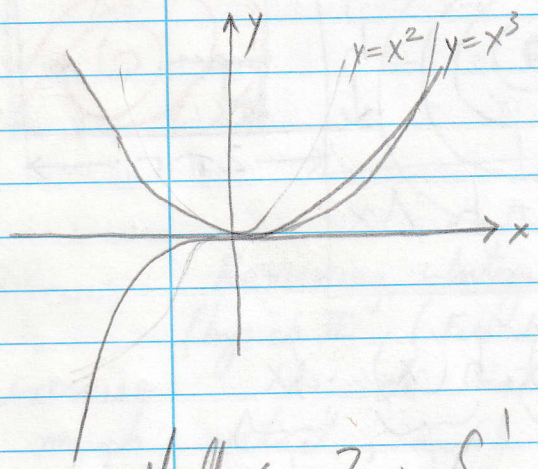
$V = 2\pi \int_1^2$ (distance from axis of rotation $= x$)(height $= x^2$) dx

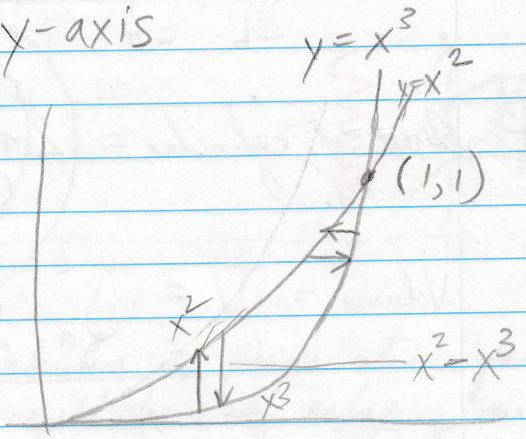
$V = 2\pi \int_1^2 x \cdot x^2 \, dx = 2\pi \int_1^2 x^3 \, dx$

$= \left(2\pi \frac{x^4}{4}\right)\Big|_1^2 = 2\pi 4 - 2\pi \frac{1}{4}$

$= 2\pi\left(4 - \frac{1}{4}\right) = \pi\left(8 - \frac{1}{2}\right) = 7.5\pi = \boxed{\frac{15\pi}{2}}$

$y = x^2$ $y = x^3$ y-axis





shells: $2\pi \int_0^1 r(x)\, h(x)\, dx$

$r(x)$: DISTANCE FROM AXIS OF ROTATION

$h(x)$: HEIGHT

$r(x) = x$: radius
$h(x) = x^2 - x^3$: height

$V = 2\pi \int_0^1 x(x^2 - x^3)\, dx = 2\pi \int_0^1 x^3 - x^4 \, dx$

$= 2\pi\left(\frac{x^4}{4} - \frac{x^5}{5}\right)\Big|_0^1 = 2\pi\left(\frac{1}{4} - \frac{1}{5}\right) = 2\pi\left(\frac{5}{20} - \frac{4}{20}\right)$

$= \pi/10 = 0.314159$ cubic units

14th: 2230 While taking a break from my writings at Barnes and Noble I came across Emil Cioran's On The Heights of Despair! They have had it only since April. I was the first to spot it — I also ordered The Trouble With Being Born It will be in in about one week.

Cioran toys with poisonous and lethal thoughts. It is so eerie for me to have at last in my possession the work of a writer I have been trying to track down since before I even discovered Schopenhauer!

I think I may begin writing pseudo-headings above entries. I have been searching for over an hour to discover precisely when I first read Emil Cioran's The Trouble With Being Born. I know it was at the Monmouth County Library Headquarters, and I also know that that book is no longer on their shelves.

While one good thing came from the search, namely — I transported all my notebooks / Logbooks / memoirs up to my lot, I am still on the trail — the Cioran trail.

It was a heading in L31 (the original Meditations of a Hermit: "Qualitas Occulta") "F∫B (2634) PLANS TO READ BOOKS BY CIORAN In there I wrote, "I had read The Trouble With Being Born before I discovered Schopenhauer..."

In L23 (Meditations of a Hermit: Diary of a Madman) the notebook which records the death of my Grandfather Hentrich (January 24th, 1991) I noticed I became a full fledged Schopenhauer disciple in April 1991 — four months after my grandfather's death. I wrote "I believe I am on to something deep here".

I really am coming around full circle This is like an LSD trip, a psychadelic experience on water... trippin' on H_2O

SA 29 MAY

searching notebooks prior to my discovery of Schopenhauer for notes about my discovery of CIORAN.

I found a trace of Cioran's influence in L20 (A Dialogue With The Inward Forces) on the 20th of April 1990: "This morning I finished reading Cioran's The Trouble With Being Born over 3 cups of coffee, cereal, and some cigarettes." Nine years ago! It seems this circle / "coming around" is larger than I thought. It is a spiral!

I am coming around a full circle from May 1998, another from September 1994 (Calculus I), and still another — April 1990 (Cioran). The day I finished that book I began reading The Outsider by Colin Wilson! I was very much a student of the occult back in 1990; hence my current interest of The Nazis and The Occult is another full circle.

I am amazed with the psychadelic quality life has — perhaps the "almost full" moon has something to do with it. At least, now that I have tracked down the initial discovery of Cioran's writings (1990), I can put my old notebooks back in the chest and start to unwind.

I want to look over my last couple interludes before settling down to read Cioran's On The Heights of Despair.

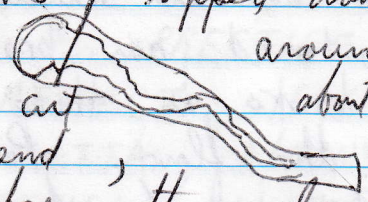
X

5:00 I have been pecking away at the Javascript manual this morning. I think I will wait until the Summer semester officially begins before I start going through Front Page 98 with the manual for COMP 99C.

I also want to sit back and relax with On The Heights of Despair. From now until June 6th I will be spontaneous: read, write Interludes, organize.

173. 23:30 On my way to college tonight, while sitting in traffic on rt 520 (I don't usually take this road & nor will I take it again), my car overheated I ignored it — but within 60 seconds the hose to the water pump blew. Stranded. Luckily I was thinking fast. I immediately pulled off the road onto a side street (in a rich neighborhood (what other kind of neighborhood are there in Lincroft/Marlboro?)).

After over an hour a struggling stubborn with the hose, I had managed to bandage it up with duct tape and electrical tape. Unbelievably, the vehicle made it to the campus!! To have seen the hose is to be in awe of this fact. It was cut ripped down the middle and all around the one end. I had to cut about 3/8 to 1/2 inch off the ripped end, and extend the tape enough for the clamp to have something to grab.



Not only did I NOT have to call a tow truck, but the vehicle made it back to Freehold after class was over (I was an hour and a half late — I got nothing out of it except a little respect from Mrs. Parking Thing (TRISH)).

I will just have to devote some serious time to Chapter 3, even if this means putting Fuzzy Logic on hold for a couple weeks.

Recap what was said on the bottom of the previous page (125): This was/is EVIDENCE OF AN ANIMAL WITH STRONG TENDENCIES TO SURVIVE AND ADAPT. This brain-body-organism was able to utilize what materials were in the trunk to handle the obstacle. Tomorrow I will drive, yes drive, to the VW dealer to purchase the hose.

1999 174 23:00 Funny (?) Story: While getting a sub at Sorento's I had a minor episode of psychosis rage. Short version: While moving along in line, the owner, with a shit eating / asshole grin tells me "Woa... you have to order back here..." when I go to place my order, he says — "just a second, hold on a minute — just chill there for a minute". Busying himself in his idiotic manner, I with that fucking smirk one wants to hit with a brick, I wait patiently. Finally, he says, "Now, can I help you?"

I ask for ½ an "Italian sub with hot peppers." He says — with a smart ass attitude, like I am some kind of jerk — "Half an Italian sub? 1 to 15." (In other words, he was putting me on the spot, forcing me to choose one of the 15 predefined subs on his god damned ~~pussy~~ menu.) I saw red for a split second as my fine tuned brain was reading loud and clear his prejudice and hatred. For some reason, perhaps he knew of me through the grapevine... the guy who used to stalk Allison Gray, the guy that had his "park sanctuary" overrun by crack heads, the guy who was pumping Mary Moss, the anti Semite who is deranged, etc...

Anyway — with that one comment, in combination with my vibes, I saw red. I turned around without a word, and I walked out with jackass saying "Sir?" behind me. Talk to my back you cocky, stupid jackass!

X

Now I am NOT seeing red. Now I am "doing math". I love math. While doing math I am a million miles away from "public opinion".

So, take that lithium. It may slow me down; and, after all — this could be half the problem in the first place. I move too fast. I get caught in the whirlwind.

I calm down when I make it back to my computer, back to my books. Why am I such a hostile human organism when I am out in public? Hypersensitive, hypertrophied consciousness, etc... How are people able to keep their cool?

Wild animal. I feel like a wild animal, a red hot alien intelligence straight out of the interstellar jungle. Here I am, trying to get Charity Care so as to pay for my psychiatric treatment.

I am a god damned mental case! I feel like I am, and I have the documents to prove it.

How will I function in the industry? I hope I don't have to interact too much with people. That is another saga altogether.

I don't mind the low frustration tolerance too much. It helps me appreciate my diagnosis. I am always close to embarassing myself. Why do I isolate? Because I do not like to AGGRAVATE the beast. I keep to myself, and I am calm in solitude.

So, tomorrow I will try to drop off the 3/13 bank statement... Such bureaucratic red tape bullshit! Fuck you! What would I do without Charity Care? I would stop treatments. I would let it fly. Will I get any FP98 work done today?

176. 22:30 Discrete Mathematics: Notes Worthy of Logbooks

Note #

0 Philosophers (ancient) were the original mathematicians

0.1 They called themselves LOGICIANS.

0.11 Logic gives a precise language for stating mathematical ideas. Ambiguity is built into our every day languages, leading to misunderstandings and incorrect conclusions.

0.2 The fundamental building block of logic is the proposition.

defn: a statement that can either be true or false $\{0, 1\}$

0.21 math note → fuzzy propositions have a degree of truth, between 0 and 1, inclusive: $[0, 1]$

0.22 proposition: $2+1=5$ is F

0.23 $x+y=9$ is a CONDITIONAL statement,

0.3 variables

0.31 the usual variables for algebra: x, y, z, etc

0.32 the logic variables: p, q, r, s, t, etc

0.4 negation

symbol ¬ ¬p means (NOT p)

0.5 connectives

0.51 conjunction (AND) → $p \wedge q$

0.52 disjunction, injunction (OR) $p \vee q$

0.53 exclusive OR (XOR) $p \oplus q$

0.6 implications

"if p then q" $p \rightarrow q$

If we start with a false hypothesis, and conclusion can be drawn.

input	p	q	p → q
	T	T	T
	F	T	T
	T	F	F
	F	F	T

0.7 converse / contrapositive (used for proof)

given implication $p \to q$

converse: $q \to p$

contrapositive: $\neg q \to \neg p$

"I think, therefore I am"

p = "I think" q = "I am"

converse: "I am, therefore I think" FALSE

contrapositive: "I am NOT, therefore I do not think." TRUE

If you can prove the contrapositive, you have proven the original proposition.

0.71 BICONDITIONAL: p if and only if q (iff)

$\leftrightarrow$

p iff q means True when $[(p \to q) \wedge (q \to p)]$

Notice how the Biconditional catches the True implications that were derived from false hypotheses:

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$ ($\wedge$, if and only if)
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

0.72 Contradictions outcome always False $(p \wedge \neg p)$

0.73 Tautology outcome always True $(p \vee \neg p)$

0.74 Logically equivalent $(\Longleftrightarrow)$

$p \Longleftrightarrow q$ if $p \leftrightarrow q$ is a TAUTOLOGY

0.8 some LOGIC exercises

tree for $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

p	q	r	¬p	¬q	¬p → ¬q	¬q → ¬p	q → r	r → q	q ↔ r	(¬p ↔ ¬q) → (q ↔ r)
T	T	T	F	F	T	T	T	T	T	T
T	T	F	F	F	T	T	F	T	F	F
T	F	T	F	T	T	F	T	F	F	T
T	F	F	F	T	T	F	T	T	T	T
F	T	T	T	F	F	T	T	T	T	T
F	T	F	T	F	F	T	F	T	F	F
F	F	T	T	T	T	T	T	F	F	F
F	F	F	T	T	T	T	T	T	T	T

$(\neg p \leftrightarrow \neg q)$

$\wedge$

$(q \leftrightarrow r) \rightarrow (\neg p \leftrightarrow q)$

T
T
T
F

$(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

T
F
T
F

Note: the number
of rows is determin

T		F
F		F
T		T
T		F
T		T

Note: the number of rows is determined by # of antecedents (p, q, r) power of 2

2^3 : p, q, r

note that the simple propositional statement $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$ includes many nested implications. The biconditional is especially tricky. We have to break it down, and work out each implication at its most basic level. These smallest basic definitions of LOGIC are AXIOMS. We build THEOREMS with these basic building blocks.

Even just the right side, $\neg p \leftrightarrow \neg q$ requires we work through p, q, $\neg p$, $\neg q$, $\neg p \rightarrow \neg q$, $\neg q \rightarrow \neg p$. To evaluate $\neg p \leftrightarrow \neg q$, we evaluate $[(\neg p \rightarrow \neg q) \wedge (\neg q \rightarrow \neg p)]$. We do the same for the right side; and then we have to evaluate $[(\neg p \leftrightarrow \neg q) \rightarrow (q \leftrightarrow r)] \wedge [(q \leftrightarrow r) \rightarrow (\neg p \leftrightarrow \neg q$

There is alot going on there. The method implemented breaks it down into a step by step process, evaluating basic implications one at a time.

0.9 Quantification

$\forall x$ for all of x
$\exists x$ there exists

$$\lim_{x \to a} f(x) = L$$

$$\forall \epsilon \, \exists \delta \, \forall x \left(0 < |x - a| < \delta \rightarrow |f(x) - L| < \epsilon \right)$$

from Calculus, expressed with Quantifiers.

example of how to read notation:

$Q =$ "$x + y = z$"

$\forall x \forall y \exists z \; Q(x, y, z)$

$\exists z \forall x \forall y \; Q(x, y, z)$

$\neg \left(\forall x \, P(x) \right)$

$\exists x \, P(x)$

$\left(\exists x \neg P(x) \right) \Longleftrightarrow \neg \left(\forall x \, P(x) \right)$

$\left(\exists x \neg P(x) \right) \Longleftrightarrow \neg \left(\forall x \, P(x) \right)$

one more time: There exists at least one x that is NOT in $P(x)$ is the logical equivalent of ~~the~~ NOT For all x in $P(x)$

$$\left(\exists x \neg P(x) \right) \Longleftrightarrow \neg \left(\forall x P(x) \right)$$

Very tired...

a better way to write this is another meaning:

$$\neg(\exists x P(x)) \iff \forall x \neg P(x)$$

NOT (There exists an x in P) is equivalent to For all of x NOT in P. ???

1. SETS

1.1 roster notation.

natural numbers, $\mathbb{N} = \{0, 1, 2, \ldots\}$
integers, $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$\mathbb{Z}$ comes from German word for ~~integers~~ number ZAHL. // my note

rational numbers, $\mathbb{Q} = \left\{ \frac{p}{q} \,\middle|\, p \in \mathbb{Z} \text{ and } q \in \mathbb{Z} \text{ and } q \neq 0 \right\}$

notation | "such that"

$\in$ "is an element of"

real numbers, $\mathbb{R} =$ { the rationals and all the stuff in between }

empty set $\emptyset = \{\}$

$A \subseteq B$
A is subset of B $\left\{ \forall x \left(x \in A \rightarrow x \in B \right) \right.$

$A \subset B$
(proper subset)

$$\left(\forall a \in A \rightarrow a \in B\right) \wedge \left(\exists b \in B \wedge b \notin A\right)$$

2 sets $A = B$ if $(A \subseteq B) \wedge (B \subseteq A)$

1.2 Cardinality

$|A|$ $\quad$ card(A) → "how many elements in the set?"

1.21 2 different types of infinity

$|A| = +\infty$ uncountably infinite

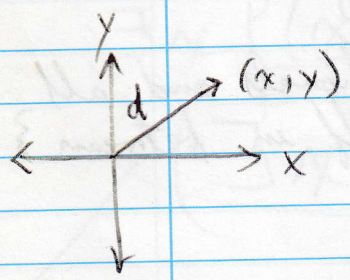
$|A| = \aleph_0$ (aleph-null)

↑ first letter of Hebrew alphabet

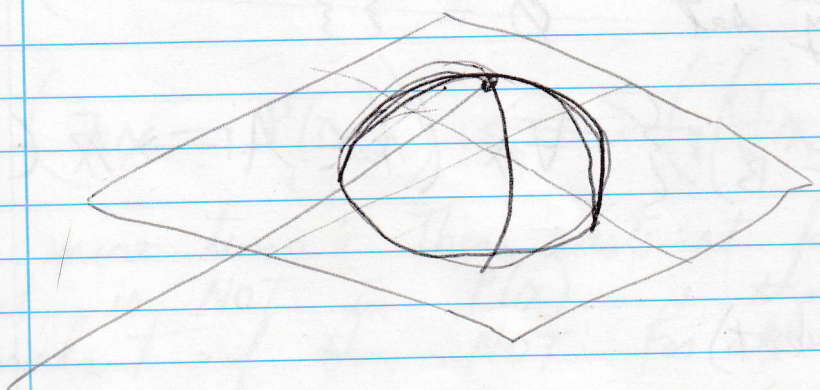
countably infinite

such as rational numbers

1.3 Visualizing Infinity (a "place")



distance from origin = d



1.4

1.5

1.4 examples

$Q(x,y) =$ "$x+y = x-y$" $\{x, y \mid \mathbb{Z}\}$

$Q(2,0) = T \quad \forall y\, Q(1,y) = F$

$\exists y \forall x\, Q(x,y) = T$

$\forall y \exists x\, Q(x,y) = F$

1.5 Power set: given set S, its Powerset $P(S)$ is the set of all subsets of S.

if cardinality of $|S|$ is n, cardinality of $P(S)$ is 2^n

if $\text{card}(S) = |S| = n < \infty$
then $\text{card}(P(S)) = 2^n$

$S = \{1, 2, 3\}$

$P(S) = \{\{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{\}, S\}$

1.6 ordered pairs, cartesian product

$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$

$A = \{1, 2\} \qquad B = \{3, 4, 5\}$

$A \times B = \{(1,3), (1,4), (1,5), (2,3), (2,4), (2,5)\}$
"A cross B"

$A \times B \neq B \times A$

$B \times A = \{(3,1), (3,2), (4,1), (4,2), (5,1), (5,2)\}$

$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x,y) \mid x \in \mathbb{R} \wedge y \in \mathbb{R}\}$

1.61 ordered n-multiples and ∞-tuple Hilbert Space

$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x,y,z) \mid x \in \mathbb{R} \wedge y \in \mathbb{R} \wedge z \in \mathbb{R}\}$

has a dimension and is countable : Taylor Series

1.7 set arithmetic $\cup \to \vee$

$\cap \to \wedge$

$\overline{A} \to \neg$

$A \setminus B \to A - B$

$$A \setminus B = \{x \mid x \in A \wedge x \notin B\}$$

$$= \{x \mid x \in A \wedge x \in \overline{B}\}$$

$$= A \cap \overline{B}$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

1.71 Generalized Unions and Intersections

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \dots \cup A_n$$

$$= \{x \mid x \in A_i \text{ for some } i\}$$

$$= \{x \mid \exists i \; x \in A_i\}$$

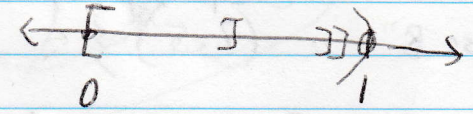
example $\bigcup_{i=1}^{\infty} [0, 1 - \frac{1}{i}] = [0, 1)$

$A_1 = [0, 0]$

$A_2 = [0, \frac{1}{2}]$

$A_3 = [0, \frac{1}{3}]$

$A_n = [0, 1)$



Space

R3

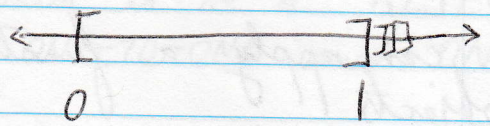
Seq

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \dots \cap A_n$$
$$= \{ x \mid \forall i \; x \in A_i \}$$

example:

$$\bigcap_{i=1}^{\infty} \left[0, 1 + \frac{1}{i}\right] = [0, 1]$$

$A_1 = [0, 2]$
$A_2 = [0, 3/2]$
$A_3 = [0, 4/3]$
$A_4 = [0, 5/4]$



$A_n = [0, 1]$

2 Functions

$f: A \to B$

"f maps elements of A to elements of B"

$f: D \to R$

A is the domain of f $\quad D(f) = A$

B is the codomain of f $\quad C(f) = B$

The range, R, is all POSSIBLE INPUTS. whereas the codomain is the actual image of the domain.

if $f(a) = b$, then b is the image of a, and a is the pre-image of b.

There may be some b's that do NOT get produced. They are in the range but not in set B — the codomain, the set of images produced by the function from its domain of input values $\forall a \in A$!

2.1 One to One and Onto

2.11 $f(n) = 2n+1$ $\quad R(f) = \{\text{odd integers}\}$

$(\mathcal{D}(f) = \mathbb{Z}, \; C^o(f) = \mathbb{Z})$

$f(n) = n^2 \quad R(f) = \mathbb{Z}^*$

$\mathbb{Z}^*$ means NON-Negative integers

2.12

we can apply a function to a whole set of objects

$f(n) = n+1 \quad \mathcal{D}(f) = \mathbb{Z}$

$S = \{0, 1, 3, 5, 7\}$

$f(S) = f(\{0,1,3,5,7\})$

$= \{f(0), f(1), f(3), f(5), f(7)\}$

$= \{1, 2, 4, 6, 8\}$

2.2 a function being ONE-TO-ONE (1-1):
when does a function have an inverse?
What is the nature of an inverse?
all legit algorithms are reversable.

ONE-TO-ONE $\longrightarrow$ INJECTION

if and only if

if $f: A \to B$ then f is 1-1 iff

$f(x) = f(y) \xrightarrow{\text{implies}}$ that $y = x$ for every $x, y \in A$.

$f(x) = f(y) \longrightarrow x = y$

contrapositive: 1-1 iff $x \neq y \longrightarrow f(x) \neq f(y)$

2.3 ONTO (SURJECTION)

$f: A \to B$ is ONTO if and only if

$\forall b \in B \quad \exists a \in A$ such that $f(a) = b$

this implies that $C_o(f) = R(f)$
the codomain equals the range.

2.4 a function is said to be a BIJECTION if it is both ONE-TO-ONE and ONTO.

If $f: A \to B$ for $a \in A \quad f(a) = b$

then the inverse image of b should be

the unique $a \in A$ such that $f^{-1}(b) = a$

If f is a bijection, then f^{-1} exists.

2.41 To find inverse of f, switch x and y and then solve for y again.

example $y = 2x + 1 = f(x)$

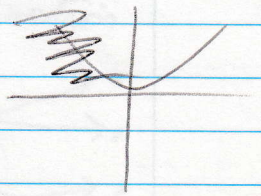
$x = 2y + 1$

$2y = x - 1$

$y = \frac{1}{2}(x-1) = f^{-1}(x)$

2.42 $f(x) = x^2 \quad (f: R \to R)$ is NOT invertible

We have to carefully define the domain and the codomain.



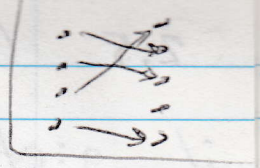
$f(x) = x^2$ & $D(f) = [0, \infty)$
$C_o(f) = [0, \infty)$

$f: [0, \infty) \to [0, \infty)$ is invertible

$f^{-1}(x) = \sqrt{x}$

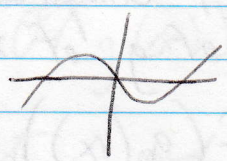
Discrete Mathematics Notes Session #2

2.43 more on 1-1 and onto



consider $f: \mathbb{R} \to \mathbb{R}$. $f(x) = x^3 - x$

$f(1) = 0$
$f(-1) = 0$

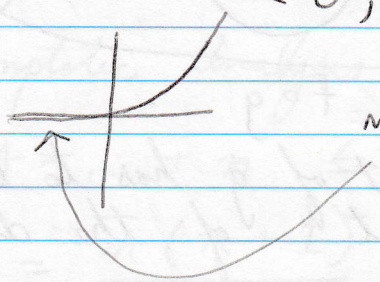


every x has a corresponding y

but not 1-1 as $x_1 \neq x_2$ does not imply $f(x_1) \neq f(x_2)$

1-1 must pass both horizontal and vertical line tests.

consider $f(x) = \begin{cases} x^2, & x \geq 0 \\ 0, & x < 0 \end{cases}$ $f: \mathbb{R} \to \mathbb{R}$

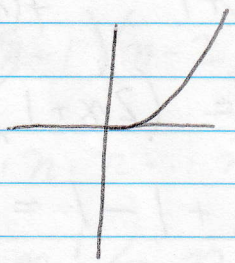


NOT 1-1

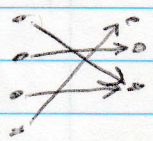
neither is x^2 with no restrictions

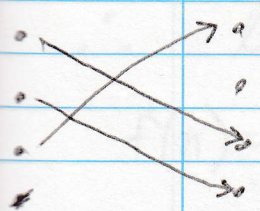
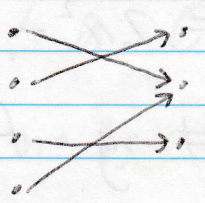
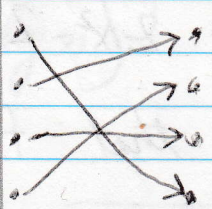
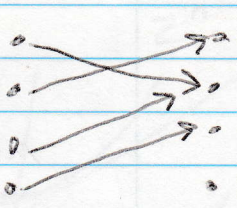
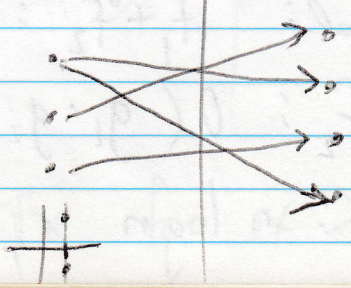
for onto, restrict the codomain ($f: \mathbb{R} \to \mathbb{R}^*$)

$f(x) = x^2$ $f: \mathbb{R}^* \to \mathbb{R}^*$ for $x \geq 0$



ONTO



1-1 NOT ONTO	ONTO NOT 1-1	1-1 and onto	Neither 1-1 OR ONTO
			
if there is no a to produce $\exists b \in B$.	here, $a_1 \neq a_4$, but $f(a_1) = f(a_4)$. why ONTO? $\forall b \in B \ \exists a \in A$ with $f(a) = b$	NOT a function 	

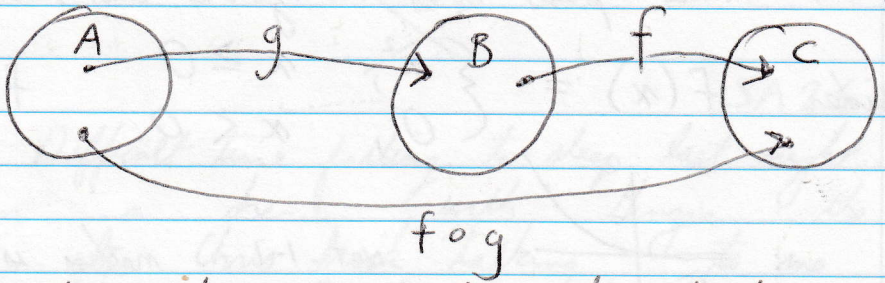
2.5 Compositions of Functions

if $g: A \to B$ $f: B \to C$ then $(f \circ g)(x)$

$f \circ g$ reads "f composed with g of x"
or "f of g of x"

$(f \circ g)(x) \to f(g(x))$

"output of $g(x)$ becomes input for f"



$f \circ g$

note: the output of g has to be at least a subset of the domain of f.

$R(g) \subseteq D(f)$ this assumption is written into the defn of $f \circ g$

quick example: $g(x) = 2x+1$ $f(x) = x^2 - 1$

$f \circ g = f(g(x)) = (2x+1)^2 - 1$

$= 4x^2 + 4x + 1 - 1 = 4x(x-1)$

$g \circ f = g(f(x)) = 2(x^2-1) + 1$

$= 2x^2 - 2 + 1 = 2x^2 - 1$

note that $f \circ g \neq g \circ f$

2.6 Growth of Functions

In general: $f_1 + f_2 : O(\max(g_1, g_2))$

$f_1 \cdot f_2 : O(g_1 \cdot g_2)$; $\log n : O(n)$ note $O(\sqrt{x})$ beta

$\log n! : n \log n$; nth degree polynomial: $O(x^n)$

2.61 little o notation

$f(x)$ is $o(g(x))$ if $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$

2.62

a function $f(x)$ is $O(g(x))$ if $\exists C$, where C is constant, and constant k such that $|f(x)| \leq C|g(x)|$ for every $x > k$.

" f is of the order of g "
" f is "dominated by" g "

2.63 Asymptotic Analysis

$1 + 2 + 3 + 4 + \ldots + n \leq n + n + n + \ldots + n$

$$f(n) = \frac{n(n+1)}{2} = \frac{1}{2}(n^2 + n)$$

$f(n) = n!$

$n! = n(n-1)(n-2) \cdots 2 \cdot 1$

$n! \leq n \cdot n \cdot n \cdots n = n^n$

$n! \leq n^n$

$\log n! \leq \log n^n$

$\leq n \log n$

$\therefore$ $\log n!$ is $O(n \log n)$

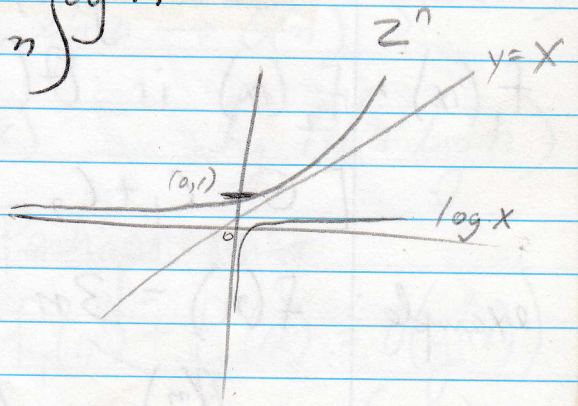
$f(n) = \log n$ is $O(n)$

$n < 2^n$

$\log n < n \log 2$

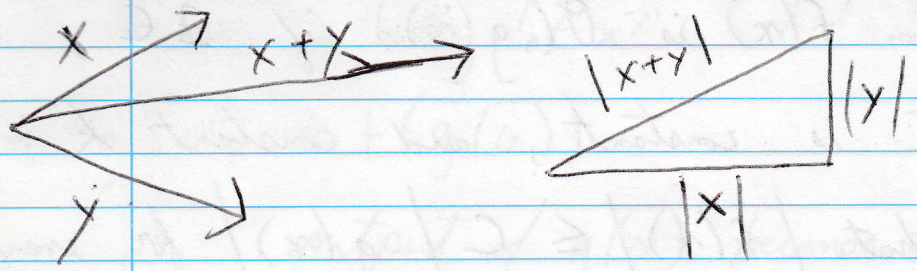
$f(n)$ $g(n)$ C

actually $\log x \leq \sqrt{x}$ for $x > 0$



2.6 More Asymptotic Analysis

$$|x+y| \leq |x| + |y|$$



What is the order of $f_1(x) + f_2(x)$?

$$|f_1(x) + f_2(x)| \leq |f_1(x)| + |f_2(x)|$$

By definition $|f_1(x)| \leq C_1 |g_1(x)|$ for $x > k_1$,

and $|f_2(x)| \leq C_2 |g_2(x)|$ for $x > k_2$

$\therefore |f_1(x)| + |f_2(x)| \leq C_1 |g_1(x)| + C_2 |g_2(x)|$

for $x > \max(k_1, k_2)$

$\leq C_1 |g(x)| + C_2 |g(x)|$ where $g(x) = \max(g_1(x), g_2(x))$

$\leq (C_1 + C_2) |g(x)|$

let $C = C_1 + C_2$ $\quad |f_1(x) + f_2(x)| \leq C |g(x)|$

$\therefore$ $f_1(x) + f_2(x)$ is $\Theta(g(x))$

where $g(x) = \max(g_1(x), g_2(x))$

$f_1(x) \cdot f_2(x)$ is $\Theta(g_1(x) \cdot g_2(x))$

[$C = C_1 + C_2$ and $k = \max(k_1, k_2)$

example: $f(n) = 3n \log n! + (n^2+3) \log n$

$3n$: $\Theta(n)$; $\log n!$: $\Theta(n \log n)$; $\Theta(n) \cdot \Theta(n \log n) = \Theta(n^2 \log n)$

n^2+3: $\Theta(n^2)$; $\log n$: $\Theta(\log n)$; $\Theta(n^2) \cdot \Theta(\log n) = \Theta(n^2 \log n)$

$\therefore f(n)$ is $\Theta(n^2 \log n)$

2.65 another example before actual problems:

$$f(x) = (x+1)\log(x^2+1) + 3x^2$$

$x+1$ is $O(x)$;
$\log(x^2+1) \approx \log x^2$ is $O(x^2)$

$3x^2$ is $O(x^2)$

$\therefore$ $f(x)$ is $O(x^3)$

2.7 Problems

Bijection from $\mathbb{R}$ to $\mathbb{R}$?

(a) $f(x) = 2x+1$ 1-1 ✓, onto ✓ YES
(b) $f(x) = x^2+1$ 1-1 (NO) NO

Find the least integer n for which $f(x)$ is $O(x^n)$:

(a) $f(x) = 2x^3 + x^3 \log x$
$g(x^3)$ x^3 (3)

(b) $f(x) = 3x^3 + (\log x)^4$ $O(n^3)$

2.71 Big O estimates

① $n \log(n^2+1) + n^2 \log n$

$n \to O(n)$
$\log(n^2+1) \to \log n^2 \to 2\log n \to O(n)$ } $O(n^2)$

$f_1(x) \cdot f_2(x)$ is $O(g_1(x) \cdot g_2(x))$ why $O(n^2 \log n)$

$n^2 \to O(n^2)$
$\log n \to O(n)$

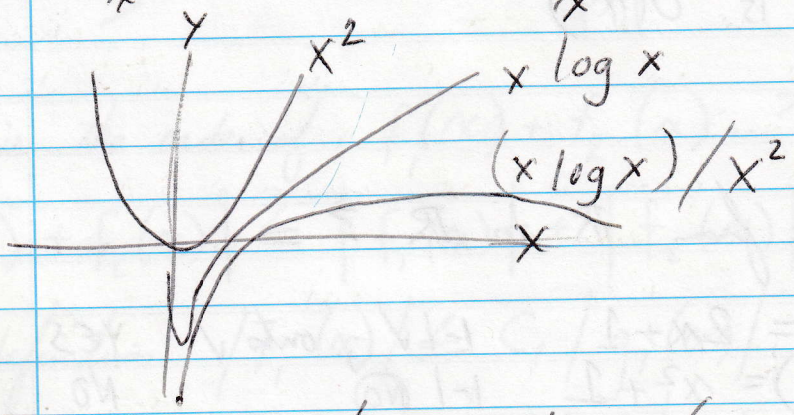
Perhaps using
$f_1(x) + f_2(x) = O(g(x))$
where $g(x) = \max(g_1(x), g_2(x))$

2.72 show that x^2 is $o(x^3)$

$$\lim_{x\to\infty} \frac{x^2}{x^3} = \lim_{x\to\infty} \frac{1}{x} = 0$$

2.73 Represent pictorially that $x \log x$ is $o(x^2)$.

$$\lim_{x\to\infty} \frac{x \log x}{x^2} = \lim_{x\to\infty} \frac{\log x}{x} = 0$$



x^2 approaches ∞ faster (grows faster).

and $(x \log x)/x^2 \to 0$

What's left? Tomorrow I will go over some homework (with transcribing it herein). I will skip the notes on Algorithms as I feel pretty up to par with that. What about Number Theory?

3. Number Theory 3.1 Theorem: if n is not prime, then it has a factor less than or equal to $\sqrt{n}$

Fundamental Theorem of Arithmetic
Let $a \in \mathbb{Z}^+$. Then a can be written as a UNIQUE product of prime numbers.

Note: If the sum of the digits of a number is divisible by 3, then the number itself is divisible by 3.

Division Algorithm: Let $a, b \in \mathbb{Z}^+$ $(b \neq 0)$ then $\exists!\ q, r \in \mathbb{Z}$ with $0 \leq r < b$ such that $a = b \cdot q + r$

q = quotient, r = remainder; a = dividend, b = divisor

1999 177 22:00

While I have been taking notes on my notes, I have been rebuilding Kant. It all started when I could not get expanded memory, mammaker, or even Galacto to work. I remember I kind of patched it all together before (see p 75) and p 77 were I copied and expanded DOS files manually.

I ran/drove over to the Barnes and Noble and read a PC repair Bible/text by Stephen Bigelow. Like a red-hot ever-mutating alien organism I scanned that book and found a Key made out of words that were abstractions of files — invisible drivers.

→ The trick was to do the following, after using On Track Disk Manager 6.03 [that's where I] go into CMOS, set disk type to "1", exit cmos with DOS BOOT, and insert Disk Manager diskette A:> DM. I know the part by heart: ADVANCED 500,250,100.]

Here's the Key: I Created a RESCUE DISK. I copied io.sys, msdos.sys, and command.com from the "DOS 6.0 boot disk onto a "clean diskette". From the Disk Manager disk I copied XBIOS.OVL and DMDRVR.BIN as well as a CONFIG.SYS file with

device = dmdrvr.bin
files = 35
buffers = 35

After DM creates new partitions, I reboot with no disks in drive A. At Western Digital's "press space bar to boot from diskette" I do so. I insert my homemade BOOT DISK — it loads XBIOS.OVL and the rest. It finishes the boot. At the A:\> prompt I insert disk 1 of the DOS 6.0 set up disks — the ones that were screaming "BOOT SECTOR VIRUS!". They took.

178 03:00 I am finished with the DM session, but I want to note here the division algorithm when it comes to negatives. Modulo always produces a positive integer.

$-24 \bmod 5 = 1$

$-24 = 5 \cdot q + r$

$-24 = 5(-5) + 1 = -25 + 1$

$-21 \bmod 4 =$

$-21 = 4 \cdot q + r$

$-21 = 4(-6) + 3 = -24 + 3$

also, $13 \bmod 4 = 5 \bmod 4 = 1$

$\therefore 13 \equiv 5 \pmod 4$

list 2 integers that are congruent to 4 mod 12.

28, 40

$28 \bmod 12 = 4$

$40 \bmod 12 = 4$

$4 \bmod 12 = 4$

$x \equiv 4 \bmod 12$ if $12 \mid (x-4)$

Show that if $a \mid b$ and $b \mid a$, where $a, b \in \mathbb{Z}$, then $a = b$ or $a = -b$

if $a \mid b \wedge b \mid a$ there exists $c, d \in \mathbb{Z}$ such that $a = bd$ and $b = ac$

in order for $a \mid b$, $\exists c \in \mathbb{Z}$ s.t. $b = ac$

in order for $b \mid a$, $\exists d \in \mathbb{Z}$ s.t. $a = bd$

if $a = bd$ and $b = ac$ then $a = acd$

if $a = acd$, then $cd = 1$?

$a = b(1) \vee a = b(-1)$

Show that if a, b, c, d are integers such that $a \mid c$ and $b \mid d$, then $ab \mid cd$.

if $a \mid c$ then $\frac{c}{a} \in \mathbb{Z}$ and $c = ax_1$

if $b \mid d$ then $\frac{d}{b} \in \mathbb{Z}$ and $d = bx_2$

$\frac{cd}{ab} \in \mathbb{Z}$

$cd = abx_1x_2$

$\therefore ab$ is a factor of cd

1999 179 MO28 JUNE

14:30 I have gotten so much satisfaction out of working on my old PC, KANT, over the past 2 days; but I was pulled away from it for most of the day yesterday as my nephew wanted me to assist him in diagnosing a machine given to his grandfather Minching by Joe, his dad (from the church). I have decided to take it slow with the Web Design class. Well, actually, I have be distracted by "the Renaissance" type passion I have for tweaking my machine. It is infused with Quality!

After rebuilding the system (program structure, drivers, config.sys, autoexec.bat, system.ini) I loaded many old 1992 styled games into drive E:. I also created my own DOS MENU SYSTEM with bat + doc files. Both versions of Derive are on C:, as well as Turbo C++ for DOS. I will continue to build it FOR FUN. There is no rush. It is a side project.

Now if I can just pull myself away from the machine for a longer period of time than it takes to smoke a cigarette, I might be able to prepare for the test. Tomorrow I will defenitely work on the Web Design stuff.

The SLACK is UP! Shall a head hunter discover me, I would be "saved" — and yet for now, I am enjoying my lack of work.

note: $\exists! q, r \in \mathbb{Z}$ with $0 \leq r < b$ such that $a = b \cdot q + r$

$-24 \bmod 5$ $\quad -24 = 5(-5) + 1$

1999 180 02:00 TU 29 JUNE

Depressed? sort of. I am having fun building up the old machine, turning Kant into an automaton of fun and games. Who knows? I may never have this much slack ever again — I will appreciate all this work when I can take a break from my work (new machine) by playing a few old style games on the old pc.

I am tempted to reread Bertrand Russles CONQUEST OF HAPPINESS

I have been getting fairly depressed lately, but it is NOT from ~~slack~~ lack of work — just lack of money/security/illusions. Besides worrying about the $666.00 I have to come up with for auto insurance (1st payment of $2400.00), I am even starting to be concerned about the possibility of my not being sought by the headhunters because of my criminal record.

I just keep hanging tough, but I can't allow myself to get too uptight — especially if I am going to get fucked over by society because of some hassles with the local authorities of this prison town!

If I don't get hired by the industry, my KNOWLEDGE still remains.

I will not commit suicide.

I will FIGHT UNI with my gnosis.

I will be a revolutionary slacker. I am devoted to learning and to adapting; but I am also devoted to slack — as most of the animals on the planet are.

Dont tell me mankind is not a cancerous insect — as a whole. Fuck it, I'm tired.

So, now, since Wednesday I have been in a mad scientist mode, working on both the old pc and Vito's "New old pc".

I even created batch files to be executed in Windows that will guide the user through creating master copies of the system disks DOS 1, 2, 3, DOS 6.2 step up, and Windows 1, 2, 3, 4, 5, 6. Another one makes a boot disk — and still another runs e:\doctor?\scan.exe /adl /clean.

After the ~~hard drive cou~~ CMOS counts its ram, Western Digital's Dynamic drive overlay takes over, and then a custom made batch file called vitomenu.bat greets the user with some classic user interfaces options: Microsoft Works version 2.0, Detect and Clean Infections, DOS Help, DOS Shell, Windows 3.1, Phone/Addresses, and the DOS prompt.

Hitting F-12 brings him back to the menu. I can't help but sense Joey gives me all the credit for this; but as much as I care about Mr. Minichini, I am too poor to even consider shelling out a few 20's. Joey's generosity and raw nerve, in combination with my determination, skill and madness brought that humble machine to heights its creators never imagined it would ever reach.

Again, I am proud of my nephew and of myself. I think Vito/Bill will enjoy the machine. I also think his daughter Kala will enjoy it — which will make him enjoy it even more.

I have been in jail. I know that a little thing like that machine can bring some wonder and enchantment into a lonely world

I can imagine how pleased I would have been to have that old, Packard Bell Legend 600X in my cell for 7 months in the county jail. I would have been able to write a book!

I will drop off some books for Joey to give his grandfather, some basic books on MS WORKS 2.0.

CLOSURE... 3AM ... a job well done. My nephew got in trouble for taking the truck. I wish he would have told me. I could have picked him up... oh well. Isn't it strange the way Joe - the father - is reacting? Is he upset that the hardware he received from the church, although valuable, just wasn't enough - that it also took some "programming" and "techno nerd" care? I put my heart into this project. My brain and my heart are one. Billy was with me. This I believe. Billy I was right here in the Mind, the spirit mind.

Let that be one word from now on. Let spirit mind → spiritmind.

Now: a smoke, then I will turn out the lights and, finally, after all this time, play RISK, the game that caused my computer to crash back in December of 1998. I think I have earned the right to move on. I can appreciate this 400 MHz machine with it's 10GB hard drive and 64 MB of RAM. I have yet to use Word Perfect 8. My cup is overflowing and yet I have no security.

1999

1999 184 15:35

New method for making entries in diaries: there is no need for continuity.

definition := What is a mathematical proof?
:= A proof is a method for communicating mathematical truth.

The flp99 gets put on hold once again, at least until I have gotten a firm grasp of mathematical proofs. I have been distracted over the past week, but now I want to settle down and get into Mathematics beyond the elementary levels.

Mathematics is the queen of science, and number theory is the queen of mathematics.

I am a little like Henry Fool from the movie of the same name.

The sum of the first n odd numbers equals n^2. More formally, this is written as

$$\sum_{k=1}^{n} (2k-1) = n^2$$

Proof by induction

basis (base case): for $n=1$, $\sum_{k=1}^{1} (2k-1) = 1 = 1^2$

induction: assume $\sum_{k=1}^{n} (2k-1) = n^2$.

We have to show $\sum_{k=1}^{n+1} (2k-1) = \sum_{k=1}^{n} (2k-1) + 2(n+1) - 1$

Using the induction hypothesis, this gives

$$\sum_{k=1}^{n+1} (2k+1) = n^2 + 2n + 1 = (n+1)^2$$

∎

The $n+1$ part goes like this:

$(2k+1)$ → $n+1$

$$\left(\sum_{k=1}^{n+1} (2k+1) \right) = \left(\sum_{k=1}^{n} (2k+1) \right) + \left(2(n+1)+1 \right)$$

1999 185 00:30 SU 04 JULY

Could I be ready for a new section already? I am actually very turned on by what is called MATHEMATICAL INDUCTION — a method of proving mathematical truths. Perhaps I may stay in this section though, as I truly experience disillusionment, when, after getting very enthusiastic about mathematics, I report to class and the professor plays the role of hot shot with his Ph.D, a doctor of philosophy in math. He has a way about him that says, "This stuff is not to be enjoyed, especially not by idiots in a community college. Math is painful. You cannot enjoy it, and whatever amount of math you think you know, you are not a mathematician unless you have a doctorate."

In other words, Mr. Thomas Bullock, as young as he is, thinks he owns mathematics! He fails to respect the fact that math is the "language of science", and that a scientist is not necessarily wearing a white lab coat, I consider myself a philosopher, a mathematician, a scientist, and a computer programmer. I don't need a degree from an institution to tell me I am "worthy". I am to this professor what the Avatar (like Jesus or the Buddha) is to the orthodox priest/Rabbi.

I care about my grade, but I care more about mathematical truth. I am jealous of his knowledge, but I am not envious. I am satisfied with my own brain — I would not trade it.

Perhaps it is time to read King's The Art of Mathematics again. Why not? I am a different man than I who read it six years ago.

In the mean time, I will continue to respect the young professor's brilliance; but this disillusionment is going to end right here. I am a pioneer. I was a pioneer when I went from Freehold to Lincroft in 1981 (CBA). I was a pioneer when I said NO to Setan Hall (ASIAN STUDIES) and worked with the "brothers" of Whistle Cleaners. Mission Mike was a true pioneer.

As a non-traditional student I am also a pioneer. Mathematics is not monopolized by the Church of Reason with its PhD's and intimidation tactics. They somewhat resemble the priests from medival times...

I am inspired by Methods of Proof and the Mathematical Induction sections. Writing the proofs will be pleasurable, not some grotesque, torturous plague!

Theorem Proving is like computer programs construction, or the other way around.
- Lemma formation is analogous to the subdivision of a large system into modules.
- Mathematical Induction accounts for the introduction of repetitive constructs such as recursion.
- Constructing well-founded relations for inductive proofs corresponds to discovering proofs of termination.

Use mathematical induction to prove that $3 + 3\cdot5 + 3\cdot5^2 + \ldots + 3\cdot5^n = 3(5^{n+1}-1)/4$ whenever n is a nonnegative integer.

Let $P(n)$ be "$\sum_{j=0}^{n} 3\cdot5^j = 3(5^{n+1}-1)/4$"

Base case: $P(1)$ is true since $\sum_{j=0}^{n} 3\cdot5^j = 3\cdot5^0 + 3\cdot5^1 = 3 + 15 = 18 = 3(5^2-1)/4 = 3(24)/4 = 3\cdot6 = 18$

SHOW THAT $P(n) \rightarrow P(n+1)$

Inductive Hypothesis: Assume that $P(k)$ is true for $k \geq 1$; then $\sum_{j=0}^{k+1} 3\cdot5^j =$

$$\left(\sum_{j=0}^{n} 3\cdot5^n\right) + 3\cdot5^{n+1}$$

$$= \frac{3(5^{n+1}-1)}{4} + 3\cdot5^{n+1}$$

$$= \frac{3(5^{n+1}-1)}{4} + \frac{12\cdot5^{n+1}}{4}$$

$$= \frac{3\cdot5^{n+1} - 3 + 12\cdot5^{n+1}}{4} = \frac{15\cdot5^{n+1} - 3}{4}$$

$$= \frac{3(5\cdot5^{n+1} - 1)}{4} = \frac{3(5^{n+2}-1)}{4}$$

Although I may sometimes feel guilty for staying up all night and working only a handful of days per month, I don't. My slack gives me mental power.

I may be a slower thinker than some, but this is my pattern in general. I am 32. The professor is 25. This implication does not imply that he groks math on a deeper level than I do.

1999

Basically, I respect why Bullock breaks the process down into 3, rather than 2 steps. There is the BASIS STEP (After further reflection I prefer Tom's "base case"). Then there is an "assume". After the "assume" — the Inductive Hypothesis — there is a "show that $P(n) \to P(n+1)$".

a simple example for clarity:

Use mathematical induction to prove the formula: Let $P(n)$ denote "$\sum_{k=1}^{n} k = \frac{n}{2}(n+1)$"

Base case: $P(1)$ is true since $\sum_{k=1}^{1} k = 1 = \frac{1}{2}(2) = 1$

$P(2)$ is true since $\sum_{k=1}^{2} k = 3 = \frac{2}{2}(2+1) = 3$

Inductive Hypothesis: $P(k) \to P(k+1)$ is true
$\forall k \in \mathbb{Z}^+$

(this is the same as: "Assume $P(k)$ is true for $k \geq 2$")

Assume $\sum_{k=1}^{n} k = \frac{k(k+1)}{2}$

Then $\left(\sum_{k=1}^{n} k\right) + (k+1) = \frac{k(k+1)}{2} + k+1$

$$\frac{k(k+1)}{2} + (k+1) = \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$

$$\frac{k(k+1)}{2} + \frac{2k+2}{2} = \frac{k^2+k+2k+2}{2}$$

$$\frac{k^2+3k+2}{2} = \frac{(k+1)(k+2)}{2}$$

which is $\frac{k}{2}(k+1)$ when $k \leftarrow k+1$

$\frac{k+1}{2}((k+1)+1)$ ▨

~~1999~~ 193 → MO 12 JUL # ~ 2451371.7 (02:00)

Plans for immediate future:

① Accept the fact that I have been in jail twice in my life, and that I do not want to become a yuppie anyway.

② Accept also that the goal of becoming a computer "programmer" is very reasonable, and that becoming a computer scientist includes mastering a considerable amount of mathematics. I am content to be a programmer as opposed to "an engineer".

③ Accept that I am broke as fuck, and realize that My wealth is my SLACK, or, more precisely — my slack is my wealth.

④ Even with the high grade point average, my chance to be truly successful in the eyes of society is grey. I have proven to be a deviant nature. All I can do NOW is continue to study computer science and hope to scrape up a little dignity in December with the realization of the Associates Degree.

This may not guarentee a good job, but a job will materialize. I will surely be granted at least that.

⑤ Stay up if I must to download SubGenius art, but tomorrow reread mathworks on proofs and browse through Fuzzy Thinking.

⑥ Reflect upon the Fall Semester, my Last Semester at Brookdale:

Physics II, Internet Programming with JAVA, and Operating Systems Technology.

⑦ Don't be afraid of all the competition out there. Besides all the mental gifts, I have the basic apparatus of a monkey/man.

I am feeling a little rebellious this twilight — a bit angry and hateful towards the "NORMALS". I am very attached to my computer — yes. I have become a true computer geek — but, alas — I have been bitten by realities too painful to allow me to glorify "marriage". I am still a horny devil who would love to jump in the sack with a honey, but I am well aware of all the COMPLICATIONS involved with my rational — emotive processes.

Angry little dick — what a wonderful name for a punk rock band.

ANGRY LITTLE DICK → (ALD)

It may reach heights, but it is anxious... I am no sex machine, although, "Doth not Nature make all animals sex machines to some extent?"

×

Why so angry? Perhaps I truly dislike people. Perhaps I have a chip on my shoulder — an attitude...

I am an unhappy man.

No affection from any woman tends to make one angry and hostile unless, of course, one manages to exert energies that would have forced Jisin from ones pipes into mental tasks. This is where the computer comes in. It is truly a MIND TOOL.

×

These background images are great. If it were not for my brainy life, I would have no life at all.

puotl → "pick up on this later"

1999.193 23:00 I returned the call I received from Jason Iverson. I will be stopping by his house Wednesday night after class. This may be cool. I have been kind of depressed lately.

Now, as far as the sensitivity I experience in MATH 226 — Jason hit on a good point. I have paid some heavy dues. I don't think there is one individual in that class who has seen the troubles I have endured. I was studying Calculus for fun in 1993. Where was professor Bullock in 1993? 6 yrs ago. He was a freshman in college studying Calculus.

So I have smoked crack and been swindled by a black con artist named Mary Moss. So I have cleaned toilets and been in the county jail twice. I think I have long since forfeited my opportunities for a clean record. I have "a past". This is reality. This is not an equation where we can "cancel out common factors" and such".

My failures define me. The kid who sits next to me in class — is he spoiled? Is he some kind of whiz kid? He seems to smirk at the amount of notes I take. Does he think I am a crack head? I don't care what he thinks. I have seen worlds within this Perfect Day that he only sees on TV. I know life itself on a deeper level.

So I am an intense thinker who does not respond well to mockery. I do not appreciate the way the instructor talks down to us. I will challenge him. I will take on the entire class!

When I present my FLP 99, I will do so with utmost sobriety. Should the smart asses mock me, I will calmly get to the point and sit down without a fuss. They are not worth it. I have nothing to prove.

There is something about me that makes me very defensive. I call it sensitivity. Dostoevsky called it a hypertrophied consciousness. The "problem" / situation is that I am intense, sensitive, and deep. I think "too hard".

Is it that I am not as smart as I thought I was? Is it that I am more intelligent than I am being made out to be by these temporary peers? There is a lack of respect in that classroom. The thought of me coming up with a theory seemed to be the brunt of a joke to the jackass next to me. and yet — I have many theories.

※

I fear the world will never take me seriously. I am an Outsider. I wish to be taken seriously, and yet the secret to happiness is to be able to laugh at oneself. I don't want to let anyone convince me that I don't measure up, but I also want to recognize that my heart beats to a different drummer. I am playing by my own rules!

1999 194 01:30 hrs JD 2451372.726 TU 13 JUL

Let there be slack.

Interesting how I find such peace of mind when I am actually engaged in working on math homework in solitude in comparison to the agitation I experience in the classroom. I will keep an eye on this. Now, perhaps the key factor is that in solitude I am fully aware that I am indeed a scholar; whereas the professor and his young fan club will not allow me to be a scholar/thinker. They want to catagorize me and view me as some kind of white trash red neck. They have no idea. Schopenhauer ... you are my mentor although you are long dead.

Now I am downloading some material on Fuzzy Logic from the internet so that I have something besides Fuzzy Thinking to drive this project I am working on. The presentation is in about 3 to 4 weeks, afterwhich I can begin to look forward to PHYSICS II. Why is there such a discrepancy between how I view myself and how I suspect I am viewed by others?

Perhaps I need a dose of Schopenhauer PAGES 55 to 124 in The Pessimist's Handbook The Wisdom of Life Position, or A Man's Place in the Estimation of Others "..."

"People generally think too much about the opinion which others form of them."

1999 195 16:30 hrs W 14 JUL

While at work, in the truck, I was able to put together several pages of notes for FLP99. Bertrand Russle is a player in the Fuzzy Revolution — the grandfather of Fuzzy Logic. Einstein did say that "so far as math is certain, it does not describe reality, and so far as math describes reality, it is not certain". I will inform Professor Bullock this evening!

I received the 2 CD's for COMPAQ Quick Restore in the mail. I ran into a problem at the formatting hard drive stage. I called for technical support... an Indian, Tomrat, was very helpful! The error message "Unable to write to drive A" was related to the CD/disk... I disabled my printer and rebooted with QR disk 1 in drive. At the first window, instead of ENTER, I chose escape.

I got an A:\> prompt!

cd system

a:\system> fdisk deleted all partitions

Power off for 10 min ~~created 1 primary DOS partition~~

1999 (MVH) 282 reboot with disk still in drive,

see p. 462

The system seems to be jammed, but it is most likely partitioning the hard drive (I hope). I will shower and give it a chance before rebooting.

196 01:30 hrs Jason was fun to visit — especially his female partner D... She is very intelligent. I really enjoyed their company. I think we may actually get together again soon ... all 3 of us ... to get some dinner or something. I really enjoyed conversing with them.

The math problem that had me confused is now under control. I have created 4 programs in my TI-85 called rCOMB, rPERM, PEVENT, ProtE.

r-Permutations $P(n,r) = n(n-1)(n-2)\ldots(n-r+1)$

$$= \frac{n!}{(n-r)!}$$

simply
```
Prompt N,R
(N!/((N-R)!))→P
Disp P
```

r-Combinations
```
Disp "N choose R:"
Prompt N,R
(N!/(R!*((N-R)!))→C
Disp "r-Combinations"
Disp "C(N,R):"
Disp C
```

Now, PEVENT, the probability that event E will occur, $P(E) = \frac{|E|}{|S|}$ can be broken down using modular programming.

$$\frac{|E|}{|S|} = \frac{C(N_1,R)}{C(N_2,R)}$$

hence

PEVENT
```
Disp "card(E):"
rCOMB
Ans→E
Disp "SET of outcomes:"
Disp "card(S):"
rCOMB
Ans→S
(E/S)→P
Disp "P(E)="
Disp P
```

Lastly, one which includes PEVENT which includes rCOMB is PnotE, which represents $P(\bar{E})$

$$P(\bar{E}) = 1 - P(E)$$
and
$$P(E) = 1 - P(\bar{E})$$

$$\frac{|S| - |E|}{|S|}$$

PnotE

```
PEVENT
1-Ans→P
Disp "P(NOT E)="
Disp P
```

example: Probability of at least 1 Ace in 5 cards

let $\bar{E}$ be probability of getting no aces in 5 cards

$$P(\bar{E}) = \frac{C(48,5)}{C(52,5)}$$

$$P(E) = 1 - P(\bar{E})$$

Lastly $P(E_1 \cup E_2) = |E_1 \cup E_2| \,/\, |S|$

$$= \frac{|E_1| + |F_2| - |E_1 \cap E_2|}{|S|}$$

$$= \frac{|E_1|}{|S|} + \frac{|F_2|}{|S|} - \frac{|E_1 \cap F_2|}{|S|}$$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

PE1UE2

```
Disp |E1| |S|
Prompt E,S
E/S→P1
Disp |E2| |S|
Prompt E,S
E/S→P2
Disp "FOR E1 and E2"
Prompt E,S
E/S→P12
Disp "P(E1UE2)="
(P1+P2-P12)→P
Disp P
```

From p. 201 some proofs

r-permutations of a set with n distinct elements is

$$P(n,r) = n(n-1)(n-2)\cdots(n-(r-1))$$
$$= n(n-1)(n-2)\cdots(n-r+1)$$
$$= \boxed{\frac{n!}{(n-r)!}}$$

The number of r-combinations of a set with n elements, where n is a positive integer with $0 \le r \le n$, equals

$$C(n,r) = \boxed{\frac{n!}{r!(n-r)!}}$$

That's why C(50,100) is a domain error!

Proof: The r-permutations of the set can be obtained by forming the $C(n,r)$ r-combinations of the set, and then ordering the elements in each r-combination, which can be done in $P(r,r)$ ways. Consequently,

$$P(n,r) = C(n,r) \cdot P(r,r)$$

This implies that

$$C(n,r) = \frac{P(n,r)}{P(r,r)} = \frac{n!/(n-r)!}{r!/(r-r)!} = \frac{n!}{r!(n-r)!}$$

The following corollary is helpful in computing the number of r-combinations of a set.

Let $n, r \in \mathbb{Z}^*$ with $r \le n$

Then

$$C(n,r) = C(n,n-r)$$

$$C(10,4) = C(10,6)$$

clearly —

$$\frac{10!}{4!6!} = \frac{10!}{6!4!}$$

198. 19:00 hours

I discovered in Explorations with TI-85 that my TI-85 has a permutations function in the MATH PROB menu.

n [nPr] r [ENTER]

and n [nCr] r enter for combinations

21:00 hrs. Some of these questions are brutal.

1999 199 02:10 hrs SU 18 JUL

My dear friend, let us make a confession. I fear I am not as intelligent as my 3.8 grade point average and National Dean's List status would have me believe. I am genuinely confused with Number Theory. Web Design is far easier!

I had planned on tackling the Fuzzy Logic Project tomorrow, but I will be lucky to get 2 pages of it written as I have been struggling all day with r-combinations. I have made a couple breakthroughs, and I am sure to ask about #'s 15 → 19 on Monday evening.

I find myself getting extremely discouraged, but I brought this upon myself. Perhaps I will want to STOP upon receiving an Associates Degree. I may find more satisfaction in applications than in theory.

For the moment I will be burning the midnight lamp and hitting these books. Poor everybody. What to do now but listen to my heart and brain. SLOW DOWN. This is not a race. Learning is sometimes painful. This stuff is not supposed to be easy

(6) # EVER MINDFUL OF DEATH

1999 199 15:00 hrs SU 18 JUL

I am fortunate to be engaged in this Discrete Mathematics course over the summer as it is keeping my brain active. My brain will be ready for Physics II come September. I feel I have a slight advantage, as a non-traditional student, over my younger classmates. I may not be as "quick and clever as ~~most~~ some, but the depth of my thought is far deeper than most. Although these last 2 sections may have been a challenge to me, I put the time into them, and I am well on my way toward completing all almost all. I will be prepared with questions come tomorrow evening.

Now, some notes... for posterity.

Binomial Coefficients

Pascal's Identity: Let $n, k \in \mathbb{Z}^+$ with $n \geq k$

Then $C(n+1, k) = C(n, k-1) + C(n, k)$

This can also be written as $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$

Proof? T is a set containing $n+1$ elements.
Let $a \in T$
Let $S = T - \{a\}$
note that there are $C(n+1, k)$ subsets of T containing k elements $\rightarrow \binom{n+1}{k}$ subsets of T containing k elements.
However, a subset of T with k elements either contains a together with $k-1$ elements of S, or contains k elements of S and does not contain a. Since there are $\binom{n}{k-1}$ subsets of $k-1$ elements of S, there are $\binom{n}{k-1}$ subsets of k

elements of T that contain a. And there are $\binom{n}{k}$ subsets of k elements of T that do not contain a, since there are $\binom{n}{k}$ subsets of k elements of S.

Note that $\binom{n}{k} \rightarrow C(n,k) \rightarrow {}_nC_k$

It is a matter of notation preferences.

Consequently

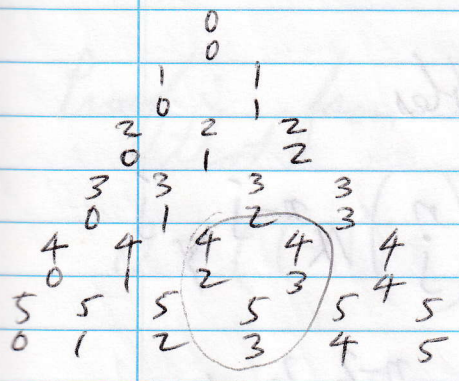
$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$

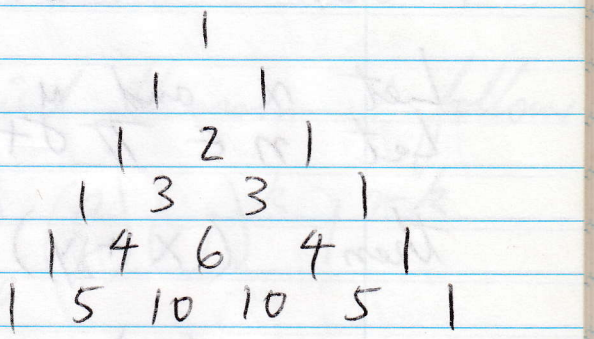
which is $C(n+1, k) = C(n, k-1) + C(n, k)$

where function $C(n,r) = n! / (r!(n-r)!)$

One "loves" mathematics at this point. I would not be taking the class just for the "fun of it" if I did not love to learn higher levels of mathematics. These summer courses, while not required, are tremendous developmental exercises.

Pascal's Triangle





$$\binom{4}{2} + \binom{4}{3} = \binom{5}{3}$$

$$\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$$

$$6 + 4 = 10$$

Pascal's identity is the basis for a geometric arrangement of the binomial coefficients in a triangle. $\binom{n}{k}$, $k = 0, 1 \ldots n$

Theorem: Let $n \in \mathbb{Z}^+$ then $\sum_{k=0}^{n} C(n,k) = 2^n$

again, this can be written as $\sum_{k=0}^{n} \binom{n}{k} = 2^n$

Proof: a set with n elements has a total of 2^n different subsets. There are $\binom{n}{n}$ subsets with n elements, $\binom{n}{1}$ subsets with 1 element, etc.

$\therefore \sum_{k=0}^{n} \binom{n}{k}$ counts the total number of subsets of a set with n elements.

This shows that $\sum_{k=0}^{n} \binom{n}{k} = 2^n$

Theorem: Let $m, n, r \in \mathbb{Z}^*$ with $r \leq m$ and $r \leq n$

then $\binom{m+n}{r} = \sum_{k=0}^{r} \binom{m}{r-k}\binom{n}{k}$

Lastly, before I return to my homework, I want to note herein the Binomial Theorem:

Let x and y be variables
Let $n \in \mathbb{Z}^+$

then $(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$

$= \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \dots$

$\dots + \binom{n}{n-1} x y^{n-1} + \binom{n}{n} y^n$

Thus, the expansion of $(x+y)^n$ can be found using combinatorial reasoning instead of multiplying the terms out.

example: $(x+y)^4 = \sum_{j=0}^{4} C(4,j)\, x^{4-j} y^j$

$= C(4,0)x^4 + C(4,1)x^3y + C(4,2)x^2y^2 + C(4,3)xy^3 + C(4,4)y^4$

$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

remember, both $\binom{n}{j}$ and $C(n,j)$ imply the same thing, namely $\frac{n!}{j!(n-j)!}$

hence $C(4,3) = \binom{4}{3} = \frac{4!}{3!\,1!} = \frac{4\cdot3\cdot2}{3\cdot2} = 4$

One last identity

let $n \in \mathbb{Z}^+$ then $\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$

Proof: from the binomial theorem it follows that

$0 = ((-1)+1)^n = \sum_{k=0}^{n} \binom{n}{k} (-1)^k\, 1^{n-k}$

$= \sum_{k=0}^{n} \binom{n}{k} (-1)^k$

note: pay little mind to the last identity ~~embrace~~ abstract. pay close attention to ~~both~~ the r-combinations notation.

Now, I would like to get a cup of java cres at Starbucks before dinner, before school; but I want to take some notes on solving linear homogeneous recurrence relations with constant coefficients.

$$a_n = a_{n-1} + 2a_{n-2}, \quad a_0 = 2, \quad a_1 = 7$$

characteristic equation:

$a_n \leftarrow r^n$: divide by r^{n-k} and equate to 0
here, $k = 2$

$$\frac{r^n}{r^{n-2}} = \frac{r^{n-1}}{r^{n-k}} + \frac{2r^{n-2}}{r^{n-2}}$$

$$r^2 - r - 2 = 0$$
$$(r+1)(r-2) = 0$$
roots: $r = -1$
$r = 2$

Hence, the sequence $\{a_n\}$ is a solution to the recurrence relation if and only if

$$a_n = \alpha_1 2^n + \alpha_2 (-1)^n$$

note $\left(a_n = \alpha_1 r_1^n + \alpha_2 r_2^n \right)$ here $r_1 = 2$, $r_2 = -1$
for some constants α_1 and α_2

From the initial conditions, it follows that

$$a_0 = 2 = \alpha_1 + \alpha_2$$

$$a_1 = 7 = \alpha_1 \cdot 2 + \alpha_2 \cdot (-1)$$

$$\alpha_2 = 2 - \alpha_1 \rightarrow \alpha_1 \cdot 2 + (2 - \alpha_1)(-1) = 7$$

$$2\alpha_1 - 2 + \alpha_1 = 7 \rightarrow 3\alpha_1 = 9 \rightarrow \alpha_1 = 3$$
$$\alpha_2 = 2 - 3 = -1$$
$\therefore$ solution is: $a_n = 3 \cdot 2^n - (-1)^n$

$$Ax = \lambda x$$

define Ax : multiply a matrix A by a column vector x ($n \times 1$ matrix)

if $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$

and $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ then $Ax = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n \end{bmatrix}$

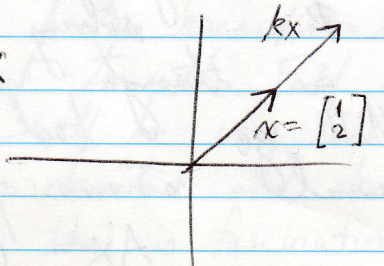
scaler multiplication $kx = \begin{bmatrix} kx_1 \\ kx_2 \\ \vdots \\ kx_n \end{bmatrix}$

example $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 4 & 0 \\ 0 & 6 & 0 \end{bmatrix}$ $x = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$

$$Ax = \begin{bmatrix} 1 & -1 & 12 \\ 2 & -4 & 0 \\ 0 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 14 \\ -2 \\ -6 \end{bmatrix}$$

Note $\bar{0}$ means a zero vector $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Find all vectors $x \neq \bar{0}$ and scalers $\lambda \neq 0$ such that $Ax = \lambda x$



$$Ax - \lambda x = \vec{0} \rightarrow (A - \lambda I)x = \vec{0}$$

We cannot subtract scaler from matrix, hence we use the Identity Matrix. It will have non-trivial solutions exactly when the determinant is $\det(A - \lambda I) = 0$

The Identity Matrix acts like 1 does in real number multiplication, it is a square matrix with ones along the diagonal and zeros everywhere else.

(2 x 2 identity): $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(3 x 3 identity): $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(4 x 4 identity): $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The letter I is used to represent the identity matrix: $IA = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 11 & 12 & 13 \\ 21 & 22 & 23 \\ 31 & 32 & 33 \end{bmatrix} = \begin{bmatrix} 11 & 12 & 13 \\ 21 & 22 & 23 \\ 31 & 32 & 33 \end{bmatrix}$$

If the result of multiplying 2 square matrices is the identity matrix, then each matrix is called the inverse matrix of the other.

$$\det(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

if $A = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$, $\det(A) = \begin{vmatrix} 3 & 2 \\ 2 & 0 \end{vmatrix} = 0 - 4 = -4$

Let $A = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$ Find eigenvalues of A:

$$\det(A - \lambda I) = 0$$

$$A - \lambda I = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 3-\lambda & 2 \\ 2 & -\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 2 \\ 2 & -\lambda \end{vmatrix}$$

$$= ((3-\lambda)(-\lambda) - 4) = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$(\lambda - 4)(\lambda + 1) = 0$$

∴ characteristic roots are $\lambda = -1, 4$
these are eigenvalues?
no, characteristic roots

Find the eigenvectors associated with/to $\lambda_1 = -1$

we want $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ such that $Ax = -1x$

$$Ax = -1x$$

$$\begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} 3x_1 + 2x_2 \\ 2x_1 \end{bmatrix} = \begin{bmatrix} -x_1 \\ -x_2 \end{bmatrix} \quad \begin{cases} 3x_1 + 2x_2 = -x_1 \\ 2x_1 = -x_2 \end{cases}$$

rewrite as $4x_1 + 2x_2 = 0$, $2x_1 + x_2 = 0$ } same equation

choose any nonzero x_1 and x_2 to satisfy the equation, such as $x_1 = -1$, $x_2 = 2$

eigenvector $x = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

eigenvalue $\lambda_1 = -1$

check → $\begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} \stackrel{?}{=} -1\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} 3(-1) + 2(2) \\ 2(-1) + 0(2) \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \checkmark$$

now use $\lambda_2 = 4$

just as before, with $A = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$, the left side of the equations are:

and the right:

$$3x_1 + 2x_2 = 4x_1$$
$$2x_1 + 0 = 4x_2$$

rewritten as $-x_1 + 2x_2 = 0$

and $2x_1 - 4x_2 = 0$

choose x_1 as 2; x_2 as 1

eigenvector $x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

eigenvalue $\lambda_2 = 4$

I may do a few problems like these hereinafter, I review Proofs, Combinatronics, and linear homogeneous recurrence relations with k coefficents, but for NOW (03:30) and for tomorrow afternoon, I will just work on eigenvalues

2.11 14:30 hrs I wonder why Bullock didn't show us how to get the determinant of 3×3's. From my own research:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{32}a_{23} - a_{12}a_{21}a_{33}$$

The 3×3 determinant consists of 3 terms. Each term contains an element of the top row multiplied by its minor.

A minor is found by crossing out all elements of its row and column, and then taking the determinant of the 2×2 matrix that is left. The signs alternate, starting with + at upper left corner.

example: $\begin{vmatrix} 2 & 7 & 4 \\ 9 & 6 & 8 \\ 5 & 1 & 3 \end{vmatrix} = 2\begin{vmatrix} 6 & 8 \\ 1 & 3 \end{vmatrix} - 7\begin{vmatrix} 9 & 8 \\ 5 & 3 \end{vmatrix} + 4\begin{vmatrix} 9 & 6 \\ 5 & 1 \end{vmatrix}$

$$= 2(6\cdot3 - 8\cdot1) - 7(9\cdot3 - 8\cdot5) + 4(9\cdot1 - 6\cdot5)$$

$$= 2(10) - 7(-13) + 4(-21) = 27$$

You don't have to expand along the first row. If a row or column has zeros, it is easier to choose it.

example:

$$\begin{vmatrix} 1 & 1 & 0 \\ 4 & 6 & 0 \\ 2 & 5 & 3 \end{vmatrix} = 0\begin{vmatrix} 4 & 6 \\ 2 & 5 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} + 3\begin{vmatrix} 1 & 1 \\ 4 & 6 \end{vmatrix} = 3(6-4) = 6$$

2/11 17:30 hrs I finished work on the eigenvalues, and now I am free for the rest of the weekend to prepare for the Discrete Mathematics exam. I may have time to rehearse FLP99, but my main objective is to go over the math at hand. I may use pencil herein. The mood has to be right. I will shower, then I will go pick up my medication. While out there I will pick up a giant cup of coffee, and then I will settle down and proclaim the mystery of my faith in mathematics. It is a beautiful phenomenon.

hypothesis implies conclusion $p \rightarrow q$

Brute Force Direct Proof

"if n is odd, then n^2 is odd."

definition of odd number $\Rightarrow 2k+1$ where $k \in \mathbb{Z}$

if $n = 2k+1$, then $n^2 = (2k+1)(2k+1) = 4k^2+4k+1$

$2(2k^2+2k)+1$

$t \leftarrow 2k^2+2k$; $n^2 = 2t+1$, where t is some integer

$\therefore n^2$ is of the form "two times an integer plus 1", which is the definition of an odd number. ▨

usually steps of a proof are algebraic

Here is another direct proof:

(i) if n is even and m is odd, then $m+n$ is odd.

n being even $\Rightarrow n = 2k$ for some $k \in \mathbb{Z}$

m being odd $\Rightarrow m = 2l+1$ for some $l \in \mathbb{Z}$

$n + m = 2k + 2l + 1 = 2(k+l) + 1$

let $t \leftarrow k+l$ where t is an integer

$n+m$ is of the form $2t+1$

$\therefore n+m$ is odd ▨

note: The definition of an odd number is any number that can be written in the form $2t+1$ for $t \in \mathbb{Z}$

note: When suspect FALSE conclusion, find 1 counter example. When suspect TRUE, must prove ALWAYS TRUE.

"if n is even and m is odd, then $n+m$ is even"
Find 1 counter example. let $n=2$ let $m=9$
$n+m=11$, which is odd
$\not\downarrow$ contradiction to the assertion.

Indirect Proof "if $3n+2$ is odd, then n is odd"

note: $p \to q \sim \neg q \to \neg p$

($3n+2$ is odd $\to$ n is odd)

$\sim$(n is even $\to$ $3n+2$ is even)

let $n = 2k$ be even
then $3(2k)+2 = 6k+2 = 2(3k+1)$
let $t \leftarrow 3k+1$ where $t \in \mathbb{Z}$
then $3n+2 = 2t$ and is even by definition.

Proof By Contradiction
"The $\sqrt{2}$ is irrational"

Proof: Assume the $\sqrt{2}$ is in fact rational.
TAKE the negation of the original conclusion, and show that we get the negation of the original hypothesis.

if $\sqrt{2}$ is rational, then $\exists\ a, b \in \mathbb{Z}$
such that $\sqrt{2} = \frac{a}{b}$

and a and b are RELATIVELY PRIME.
(to be relatively prime means to share no common factors → LCM, or lowest common terms)

Square both sides and cross multiply

$2 = \frac{a^2}{b^2}$ $\quad 2b^2 = a^2$, so a must be even

$2b^2$ is of the form $2k$ and is ∴ even, hence a^2 is even
squares of even integers are even.
squares of odd integers are odd.
a can be written as $2k$, so $2b^2 = 4k^2$; $b^2 = 2k^2$
which means b is also even.
But, if $\frac{a^2}{b^2} = \frac{a \cdot a}{b \cdot b}$ is even, then both can be divided by 2 and, hence, not relatively prime

note: if a is even, then a^2 is even can be proved.

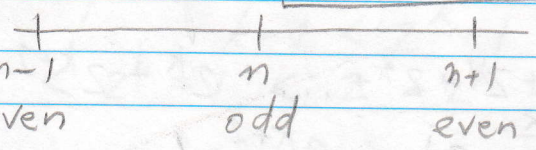
DIRECT $(2k)^2 = 4k^2 = 2(2k^2)$ of form $2t$

INDIRECT $p \to q \sim (\neg q \to \neg p)$

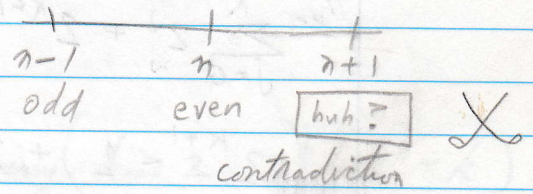
if n^2 is odd, then n is odd.

since n^2 is odd, we can write $n^2 = 2k+1$ for some integer k: $n^2 - 1 = 2k$, $(n+1)(n-1) = 2k$

case 1



case 2 $n+1$ is even, $n-1$ is odd



Mathematical Induction

Prove that the sum of the first n integers is $\frac{n(n+1)}{2}$

I. Base case true for $n = 2$

$$1 + 2 \stackrel{?}{=} \frac{n(n+1)}{2} = \frac{2(2+1)}{2} = 3 \quad \checkmark$$

II. Inductive Hypothesis

Suppose the formula is in fact true for some $k \geq 2$ and $k < n$

$$1 + 2 + \ldots + k = \frac{k(k+1)}{2}$$

This also can be written as $\sum_{j=1}^{k} j = \frac{k(k+1)}{2}$

III Show that the formula is true for $k+1$

(Using II)

$$\underbrace{1 + 2 + \ldots + k}_{\text{This much is } k(k+1)/2} + k + 1 = \sum_{j=1}^{k+1} j \stackrel{?}{=} \frac{(k+1)(k+2)}{2}$$

$$\frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{k^2+k+2k+2}{2} = \frac{k^2+3k+2}{2} = \frac{(k+1)(k+2)}{2}$$

VIII

$1 + 2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 1$

Proof I. Base case: $n = 0$

$1 = 2^{0+1} - 1 = 2^1 - 1 = 1$ ✓

II. Inductive hypothesis

assume the formula is true for some $k \geq 0$

$2^0 + 2^1 + 2^2 + \ldots + 2^k = 2^{k+1} - 1$

III Show that the formula is true for $k+1$

$2^0 + 2^1 + 2^2 + \ldots + 2^k + 2^{k+1} \stackrel{?}{=} 2^{k+2} - 1$

assume $\sum_{j=0}^{k} 2^j = 2^{k+1} - 1$

$\sum_{j=0}^{k} 2^j + 2^{k+1} \stackrel{?}{=} 2^{k+2} - 1$

$(2^{k+1} - 1) + 2^{k+1} \stackrel{?}{=} 2^{k+2} - 1$

$2(2^{k+1}) - 1 = 2^{k+2} - 1$

$2^1(2^{k+1}) - 1 = 2^{k+2} - 1$

$1 + 3 + 5 + \ldots + 2n - 1 = n^2$

PROOF

I. Base case when $n = 2$

does $2(1) - 1 = 1$? yes

does $1 + 2(2) - 1 = 2^2$ yes

does $1 + 3 + 2(3) - 1 \stackrel{?}{=} 3^2 = 9$ yes

II Inductive Hypothesis

assume the formula is true for $1 + 3 + 5 + \ldots + (2k-1) = k^2$

III. Show that the formula is true for

$1 + 3 + 5 + \ldots + (2k-1) + (2(k+1)-1) = (k+1)^2$

$1 + 3 + 5 + \ldots + (2k-1) + (2k+1) =$

$k^2 + 2k + 1 = (k+1)(k+1)$

$= k^2 + 2k + 1$

HOLY SHIT

Show that $n < 2^n$ for any $n \in \mathbb{Z}^*$ (NON-ZERO ~~positive~~ integer)

Proof I. Base Case

$n = 0 \quad 0 \overset{?}{<} 2^0 = 1$ The induction has begun

It is THAT simple!

II. Inductive Hypothesis

For some $k \geq 0$, $k < 2^k$ Again, nothing to it.

III. Show that the formula is true for $k+1$.

$k+1 < 2^{k+1}$

starting with assumption: $k < 2^k$

$k+1 < 2^k + 1$

$k+1 < 2^k + 1 < 2^k + 2^k = 2^1 \cdot 2^k$

$= 2^{k+1}$

Show that if $n \in \mathbb{Z}^+$ then $3 \mid (n^3 - n)$ $\mid \rightarrow$ "divides"

I. Base case $3 \mid (2^3 - 2)$

II Inductive Hypothesis for any $k \geq 2$ $3 \mid (k^3 - k)$

III Show that the assertion is true for $(k+1)$

does $3 \mid ((k+1)^3 - (k+1))$?

$(k^3 + 3k^2 + 3k + 1) - (k+1) = k^3 + 3k^2 + 3k - k$

$= \underbrace{(k^3 - k)}_{3 \mid (k^3-k)} + \underbrace{3(k^2 + k)}_{3 \mid 3(k^2+k)}$ $\therefore 3 \mid ((k+1)^3 - (k+1))$

1999 212 SA 31 JULY 03:45 hrs

I saw "The Blair Witch Project". It was not a movie, but a film made out of footage discovered in a rapsack buried in the cellar of an old (100 yrs) house in the Black Hills woods in Maryland. The way the film ended, the last 15 seconds, burned into my mind. It could not help but be terribly ambiguous. I believe the footage is real, unlike the local police chief who stated publically that it is a hoax. If a hoax, where are Heather, Josh, and Mike now? Primal Fear is of the dark and the unknown, not Space Aliens!

HOLY SHIT

The product of a rational and an irrational is irrational

let $q \in \mathbb{Q}$ with $q \neq 0$ and let m be irrational

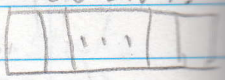
$q \to \frac{r}{s}$ where $r, s \in \mathbb{Z}$

if $m \cdot q = t \in \mathbb{Q}$ then $m = \frac{t}{q} \Rightarrow m \in \mathbb{Q}$

since $t \in \mathbb{Q}$ $\exists\, a, b \in \mathbb{Z}$ such that $t = \frac{a}{b}$
and $q \in \mathbb{Q}$ $\exists\, r, s \in \mathbb{Z}$ such that $q = \frac{r}{s}$
then $m = t/q = (a \cdot s)/(b \cdot r)$
$\therefore m \in \mathbb{Q}$ ✗

COUNTING TECHNIQUES

S is a finite set where $n = \text{card}(S)$
$P(S)$ = "powerset of S"
= "the set of all subsets of S"
$\text{card}(P(S)) = 2^n$

The question of the cardinality of a bit string has to do with the possible subset drawn.
(2^n possible ways of filling the "buckets" )

example: A is a set with m elements
B is a set with n elements
How many 1 to 1 functions are there $f: A \to B$?
$m > n$ is impossible
if $m > n$, something in the domain has to get mapped to something in the codomain more than once
$m = n$ is fine
$m \leq n$ is fine

elements of A: $a_1, a_2, \ldots, a_n$
elements of B: $b_1, b_2, \ldots, b_m$

n choices for a_1, $n-1$ choices for a_2,
$n-2$ choices for a_3, ..., $(n-(m-1))$ choices for a_m
Possible ways to define f:
$n(n-1)(n-2)\ldots(n-m+1) = n!/(n-m)!$

Count possible telephone numbers NYX NNX XXXX

where $X = \{0, 1, \ldots, 9\}$ NYX → $8 \cdot 2 \cdot 10 = 160$

$N = \{2, \ldots, 9\}$ NNX → $8 \cdot 8 \cdot 10 = 640$

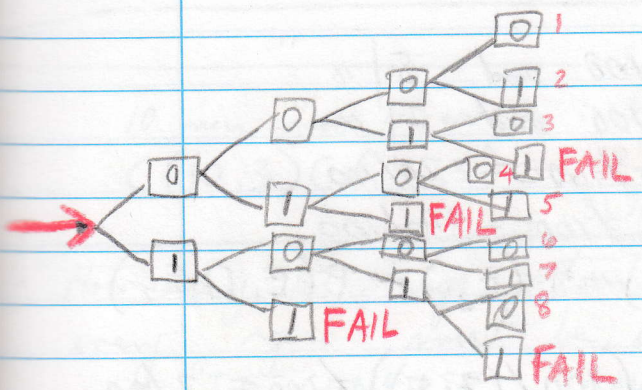
$Y = \{0, 1\}$ XXXX → $10^4 = 10{,}000$

∴ total → 102,400,000

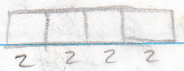
New system: NXX NXX XXXX

$10^8 \cdot 8^2 = 6{,}400{,}000{,}000$

Tree: find # of 4 bit strings with no consecutive 1's.



with math:



$2^n = |S|$

$|S| = 2^4 = 16$

but what about $|E|$?

? use tree

Some counting examples

How many different 3 letter initials exist? 26^3

my theory → "number of placeholders "^" digits"

How many 10 digit bit strings begin and end with a 1?

total bit strings $= |S| = 2^{10}$

the ones that begin with 1 are ½ of 2^{10}

$2^{10} = 2^9 * 2^1 = 2^9 + 2^9$ ∴ ½ of $2^{10} = 2^9$

of this half, half end with a one ∴ $2^9 = 2^8 * 2$

solution: 2^8

if county code is 1 to 3 digits (X, XX, or XXX) how many different numbers are available? (NXX NXX XXXX)

from above NXX NXX XXXX: $t \leftarrow 6{,}400{,}000{,}000$

then, total $= 10t + 10^2 t + 10^3 t = 6.4(10^{10}) + 6.4(10^{11}) + 6.4(10^{12})$

$= 7.104(10^{12})$

Permutations and Combinations: see p. 201 to 214

lottery: 44 numbers choose 6

$|E| = 6!$ $|S| = 44^6$ $P(E) = 6!/44^6$

example: 2 events E_1 = "diamond?" and E_2 = "picture card"

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$P(E_1 \cap E_2) = \frac{3}{52}$; $P(E_1 \cup E_2) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26}$

Problem: There are 100 tickets, numbered #1, #2, ..., #100. They are sold to 100 different people for a drawing. 4 prizes are awarded, including a grand prize.

How many ways are there to award prizes if tickets #47 and #19 are known to have won prizes?

$$C(4,1)\,C(3,1)\,P(98,2) = \frac{4!}{1!3!}\cdot\frac{3!}{1!2!}\cdot\frac{98!}{96!} = 114072$$

A club has 25 members. How many ways are there to choose 4 members of the club to serve on a committee?

$$C(25,4) = \frac{25!}{4!21!} = 12650$$

- to choose Pres, VP, Treas, Sec?

$$P(25,4) = \frac{25!}{21!} = 303600$$

10 men and 15 women: committee with 3 M and 3 F?

$$C(10,3)\,C(15,3) = \frac{10!}{3!7!}\cdot\frac{15!}{3!12!} = 54600$$

How many bit strings contains exactly 8 zeros and 10 ones if every zero must be immediately followed by a one?

"IF CONFUSED, DRAW A PICTURE" - Prof. Tom Bullock

1	01	01	01	01	01	01	01	01	1

The sets of 01's can be viewed as 1 element each, hence 8 sets of 01 + 2 sets of 1 equal 10 members. The 2 extra ones will be placed in every possible position, hence 10 choose 2 → $_{10}C_2 = C(10,2) = 45$

2/12 20:45 hrs note: I added a module in pMENU on TI-85 that does $P(E) = \sum_{i=1}^{n} P(x_i)$ where $E = \{x_1, x_2, \ldots, x_n\}$. It adds the sums of Bernoulli trials...

Another program I added is called SUCFAIL. The probability of k successes in n independent Bernoulli trials, with probability of success p, and probability of failure $q = 1 - p$ is $C(n,k)\,p^k q^{n-k}$

The code for SUCFAIL is relatively simple to follow. The code for "trials" — a module in pMENU — is a little more ambiguous as far as the TI-85 functions go.

```
ClLCD
Outpt(1,1," at least K successes)
... blah blah blah...
Prompt K,N,P
1-P→Q
```

```
sum seq(N nCr K)(P^K)(Q^
(N-K)),K,K,N,1)→
R
```

$$\sum_{K=K}^{N} C(N,K)\,P^K\,Q^{N-K}$$

2 loaded die

die 1 6 is 3 times as likely to come up.

die 2 let 1 come up $\frac{1}{2}$ as often as the others

$S_{die1} = \{1, 2, 3, 4, 5, 6\}$

let $x = p(n) \quad 1 \le n \le 5$

$p(6) = 3x$

$$\sum_{n=1}^{6} p(n) = 5x + 3x = 8x = 1 \; ; \; x = \frac{1}{8}$$

$p(6) = 3/8 \qquad p(n) = \frac{1}{8}$ for $1 \le n \le 5$

$S_{die2} = \{1, 2, 3, 4, 5, 6\}$

let $x = p(u) \quad 2 \le n \le 6$ $\qquad p(u) = \frac{2}{11}$

$p(1) = \frac{1}{2} x \qquad p(1) = 1/11$

$$\sum_{u=1}^{6} p(u) = 5x + \frac{1}{2}x = \frac{11x}{2} = 1 \; ; \; x = \frac{2}{11}$$

$S_{7+} = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$P((1,6)) = \frac{1}{8} \cdot \frac{2}{11} = \frac{2}{88}$

$P(2,5) = \frac{1}{8} \cdot \frac{2}{11} = \frac{2}{88}$

$P(3,4) = \frac{1}{8} \cdot \frac{2}{11} = \frac{2}{88}$

$P(4,3) = \frac{1}{8} \cdot \frac{2}{11} = \frac{2}{88}$

$P(5,2) = \frac{1}{8} \cdot \frac{2}{11} = \frac{2}{88}$

$P(6,1) = \frac{3}{8} \cdot \frac{1}{11} = \frac{3}{88}$

$5\left(\frac{2}{88}\right) + \frac{3}{88} = \frac{13}{88} \sim 14.7$

$E = \{e_1, e_2, \ldots, e_n\}$

$P(E) = P(e_1) + P(e_2) + \ldots + P(e_n)$

$$\sum_{i=1}^{n} P(e_i)$$

Draw a card from a well shuffled deck.
We are told the card is black.
What is the probability the card is a black jack.

with given information, $P(\text{BlackJack}) = \frac{2}{26}$

$$P(\text{BlackJack} \mid \text{card is black}) = \frac{2/52}{1/2} = \frac{4}{52} = \frac{1}{13}$$

↑ Given that

$$P(\text{card is black}) = \frac{26}{52}$$

$$P(\text{black card AND a Jack}) = \frac{2}{52}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

E = " getting an even heart" $P(E) = 5/52$
F = " drawing a red card" $P(F) = 1/2$

$E \subseteq F \quad \therefore \quad E \cap F = E$

$$P(E|F) = \frac{5/52}{26/52} = \frac{5}{52} \cdot \frac{52}{26} = \frac{5}{26}$$

Independence: if the likelihood of E occuring is not dependent on knowledge of the likelihood of F occuring, then
$P(E|F) = P(E)$

If E and F are independent, $P(E|F) = \frac{P(E \cap F)}{P(F)} = P(E)$
$* \quad P(E \cap F) = P(E) \cdot P(F)$

Proofs ✓ Combinations & Permutations ✓ next...

RECURRENCE RELATIONS

$\{a_n\}$

Numerical sequence where the nth term is determined with an equation which determines the nth term, a_n, by the terms which precede it.

$$(a_0, a_1, \ldots, a_n)$$

where $n > n_0$ and is non-negative

The solution is a sequence which satisfies the equation.

$a_n = 2a_{n-1} - a_{n-2}$ $\quad a_1 = 3_n$

$3n = 2(3(n-1)) - 3(n-2)$

$3n = 6n - 6 - 3n + 6 = 3n$ ✓

Linear, Homogeneous, with constant coefficient and degree k:

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \ldots + c_k a_{n-k}$$

where c_i are real numbers (not variables)

refer to notes on p. 217

Before getting deep into this, let us hit a few more probability problems.

Show that if E and F are events, then

$P(E \cap F) \geq P(E) + P(F) - 1$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$ and $P(E \cup F) \leq 1$ } These are THEOREMS

It follows that $1 \geq P(E) + P(F) - P(E \cap F)$

and $P(E \cap F) \geq P(E) + P(F) - 1$

Show that if $E_1, E_2, \ldots, E_n$ are event from a finite sample space, then

$$p(E_1 \cup E_2 \cup \cdots \cup E_n) \leq p(E_1) + p(E_2) + \cdots + p(E_n)$$

let $p(n)$ be "$p(\bigcup_{j=1}^{n} E_j) \leq \sum_{j=1}^{n} p(E_j)$"

base case:

$p(2) = p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$

$\leq p(E_1) + p(E_2)$

Show that this is true for $p(n+1)$

$$p\left(\bigcup_{j=1}^{n+1} E_j\right) = p\left(\bigcup_{j=1}^{n} E_j\right) + p(E_{n+1}) \leq \sum_{j=1}^{n+1} p(E_j)$$

(?)

$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$

let $a_n = x^n$ $\quad x^n = 6x^{n-1} - 11x^{n-2} + 6x^{n-3}$

$x^3 = 6x^2 - 11x + 6$

$x^3 - 6x^2 + 11x - 6 = 0$

TI-85 POLY gives
$x_1 = 3$
$x_2 = 2$
$x_3 = 1$

general solutions $a_n = \alpha_1 (1)^n + \alpha_2 (2)^n + \alpha_3 (3)^n$

$a_n = \alpha_1 + \alpha_2 2^n + \alpha_3 3^n$

imposing initial conditions $a_0 = 2$, $a_1 = 5$, $a_3 = 15$

$n=0$: $\alpha_1 + \alpha_2 + \alpha_3 = 2$
$n=1$: $\alpha_1 + 2\alpha_2 + 3\alpha_3 = 5$
$n=2$: $\alpha_1 + 4\alpha_2 + 9\alpha_3 = 15$

TI-85 SIMULT
$\alpha_1 = 1$
$\alpha_2 = -1$
$\alpha_3 = 2$

particular solution $a_n = 1 + (-1)2^n + 2 \cdot 3^n$
$= 1 - 2^n + 2 \cdot 3^n$

$f_n = f_{n-1} + f_{n-2}$ $\quad$ let $f_n = x^n$

$x^n = x^{n-1} + x^{n-2}$ $\quad x^2 = x + 1$

$x^2 - x - 1 = 0$ $\quad$ use quadratic equation

$ax^2 + bx + c = 0$ $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{1 \pm \sqrt{1 - 4(1)(-1)}}{2(1)} = \dfrac{1 \pm \sqrt{5}}{2}$

hence, general solution:

$$f_n = \alpha_1 \left(\frac{1+\sqrt{5}}{2}\right)^n + \alpha_2 \left(\frac{1-\sqrt{5}}{2}\right)^n$$

initial conditions $f_0 = 1$, $f_1 = 1$

$n=0$: $\alpha_1 + \alpha_2 = 1$

$n=1$: $\left(\frac{1+\sqrt{5}}{2}\right)\alpha_1 + \left(\frac{1-\sqrt{5}}{2}\right)\alpha_2 = 1$

solve by multiplying top (a_0) equation by $\left(\frac{-1-\sqrt{5}}{2}\right)$

↪

$$\left(\frac{-1-\sqrt{5}}{2}\right)\alpha_1 + \left(\frac{-1-\sqrt{5}}{2}\right)\alpha_2 = \frac{-1-\sqrt{5}}{2}$$

$$\left(\frac{1+\sqrt{5}}{2}\right)\alpha_1 + \left(\frac{1-\sqrt{5}}{2}\right)\alpha_2 = 1$$

$$0 - \sqrt{5}\alpha_2 = \frac{1-\sqrt{5}}{2}$$

$\frac{-1-\sqrt{5}+1-\sqrt{5}}{2}$

$\frac{-2\sqrt{5}}{2} = -\sqrt{5}$

$$\alpha_2 = \frac{1-\sqrt{5}}{-2\sqrt{5}}$$

$\frac{-1-\sqrt{5}}{2} + \frac{2}{2}$

$\frac{1-\sqrt{5}}{2}$

$$\alpha_2 = \frac{1-\sqrt{5}}{-2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}-5}{-10} = \frac{5-\sqrt{5}}{10}$$

$$\alpha_1 = 1 - \alpha_2 = \frac{10}{10} - \frac{5-\sqrt{5}}{10}$$

$$\alpha_1 = \frac{5+\sqrt{5}}{10}$$

$$f_n = \left(\frac{5+\sqrt{5}}{10}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{5-\sqrt{5}}{10}\right)\left(\frac{1-\sqrt{5}}{2}\right)^n$$

---

suppose that the characteristic polynomial has a root of multiplicity p; then that particular root z gives part of the general solution:

$$\alpha_1 z^n + \alpha_2 n z^n + \alpha_3 n^2 z^n + \ldots + \alpha_p n^{p-1} z^n$$

$\alpha_p n^{p-1} z^n$

$\alpha_1 n^0 z^n = \alpha_1 z^n$

$\alpha_2 n^1 z^n = \alpha_2 n z^n$

$\alpha_3 n^2 z^n$

$$a_n = 6a_{n-1} - 9a_{n-2}$$

$a_n = x^n \qquad x^2 = 6x - 9 \qquad x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0 \qquad x = 3$ has multiplicity of 2

general solution: $a_n = \alpha_1 3^n + \alpha_2 n 3^n$

$a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$

let $a_n = x^n$ degree 2 $x^2 = 5x - 6$

$x^2 - 5x + 6 = 0$ $(x-2)(x-3) = 0$ $x = 2, 3$

general solution:

$$a_n = \alpha_1 2^n + \alpha_2 3^n$$

$n=0$: $\alpha_1 + \alpha_2 = 1$

$n=1$: $2\alpha_1 + 3\alpha_2 = 0$

$-2\alpha_1 - 2\alpha_2 = -2$

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$\alpha_1 - 2 = 1 \rightarrow \alpha_1 = 3$

particular solution:

$$a_n = 3 \cdot 2^n - 2 \cdot 3^n$$

$a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$; $a_0 = 7$, $a_1 = -4$, $a_2 = 8$

let $a_n = x^n$ $x^n = 2x^{n-1} + 5x^{n-2} - 6a^{n-3}$

$x^3 = 2x^2 + 5x - 6$; $x^3 - 2x^2 - 5x + 6 = 0$

TI-85 POLY $\rightarrow$ $x = 3, -2, 1$

general solution: $a_n = \alpha_1 3^n + \alpha_2 (-2)^n + \alpha_3$

$n=0$: $\alpha_1 + \alpha_2 + \alpha_3 = 7$

$n=1$: $3\alpha_1 - 2\alpha_2 + \alpha_3 = -4$

$n=2$: $9\alpha_1 + 4\alpha_2 + \alpha_3 = 8$

TI-85 SIMULT $\rightarrow$ $\alpha_1 = -1$, $\alpha_2 = 3$, $\alpha_3 = 5$

particular solution: $a_n = -3^n + 3(-2)^n + 5$

1999 213 SU 01 AUG 01:30 hrs

All I really have left to go over is Graph Theory and Matrices. I will peck away at it tonight, and finish up tomorrow afternoon. Although I plan on touching up the FLP tomorrow evening, I can put off rehearsing for the presentation until next weekend after I complete the final exams. I will be relieved to make it to the morning of August 12th. I will be sure to swim in the ocean before Fall semester.

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$G = (V, E)$ is a directed graph

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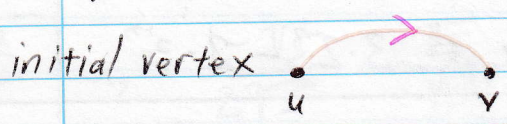

in-degree: $\deg^-(v)$ = "# of edges that have v as the terminal vertex"

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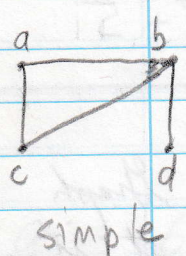
theorem: for a digraph, $\sum_{v \in V} \deg^-(v) + \sum_{v \in V} \deg^+(v) = |E|$

definition of bipartite: a simple graph in which there are two non-empty sets V_1 and V_2 such that (1) $V_1 \cup V_2 = V$ and $V_1 \cap V_2 \neq 0$

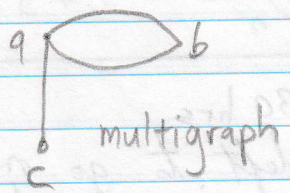
(2) all edges connect vertices in V_1 to vertices in V_2

NO INTERNAL EDGES

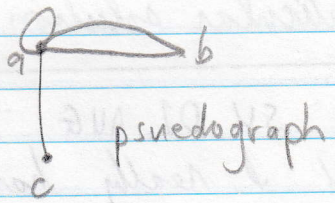
note: any graph that contains a cycle as a proper subset cannot be bipartite.



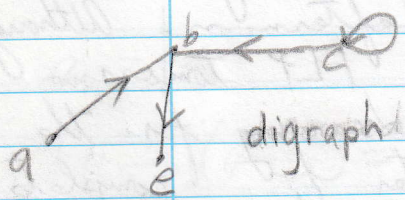
simple



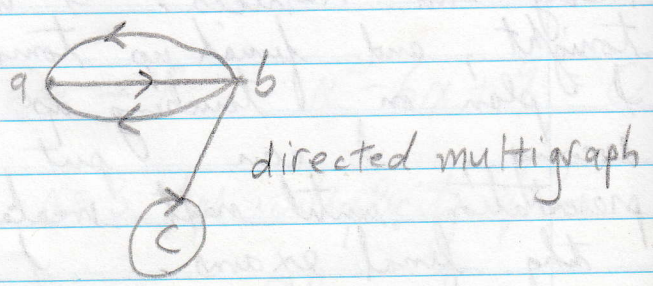
multigraph



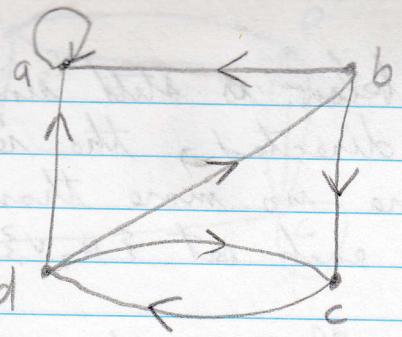
psuedograph



digraph



directed multigraph



$v = 4 \quad e = 7$

$\deg^-(a) = 3 \quad \deg^+(a) = 1$

$\deg^-(b) = 1 \quad \deg^+(b) = 2$

$\deg^-(c) = 2 \quad \deg^+(c) = 1$

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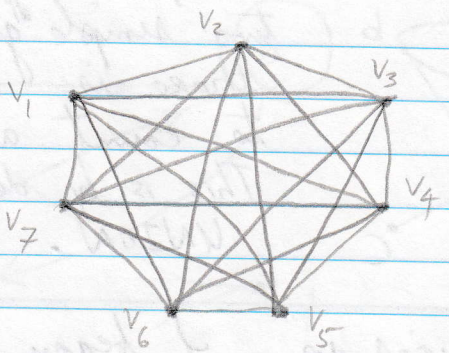
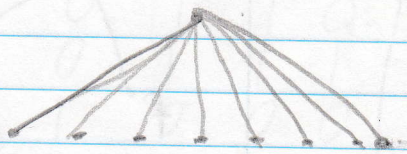
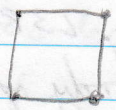
K_7



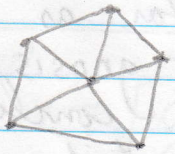
$K_{1,8}$



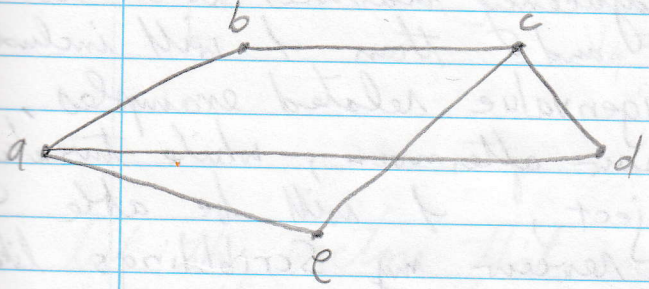
C_4

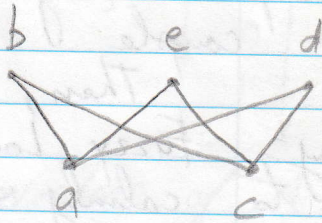


W_5



Make bipartite

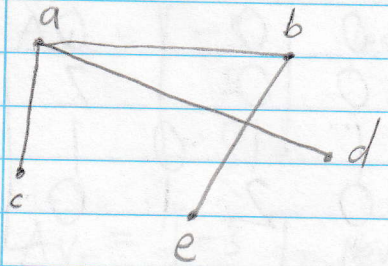




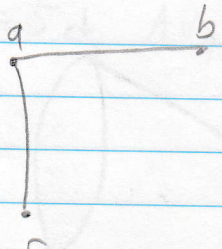
$G = (V, E)$ $\quad H = (W, F)$ is a subgraph of G if $W \subset V$ and $F \subset E$

"is contained in"

$G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$



G_1



G_2

total of 6 vertices

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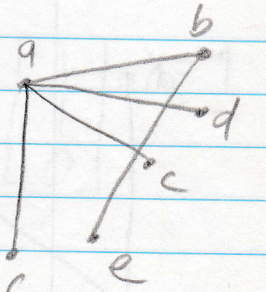

$a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$

let $a_n = x^n$ degree 2 $x^2 = 5x - 6$

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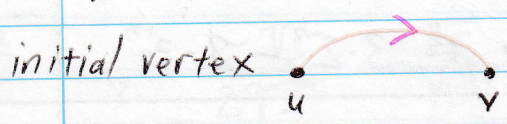

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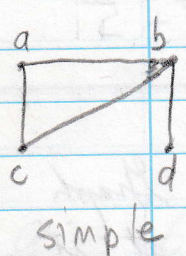

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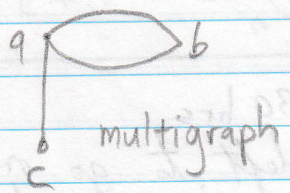
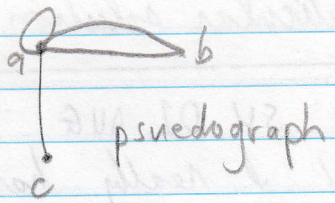


multigraph



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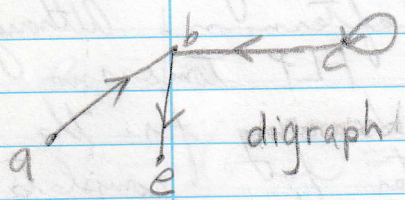

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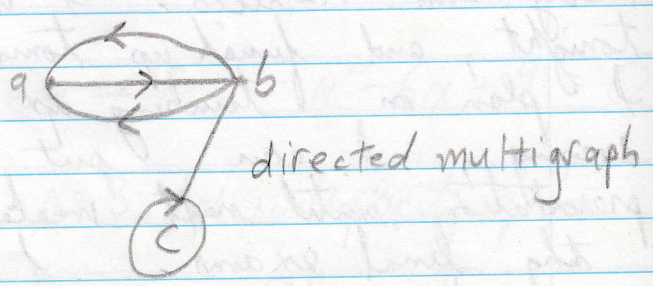
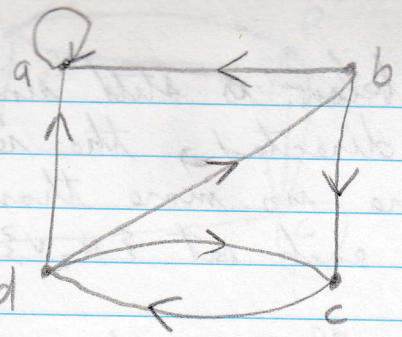


directed multigraph



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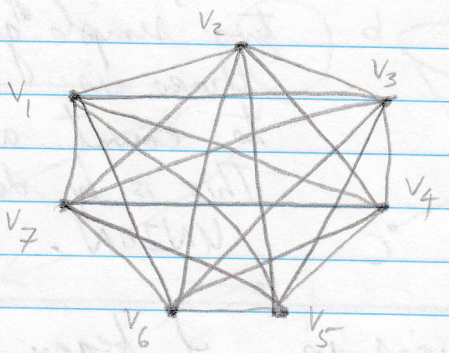
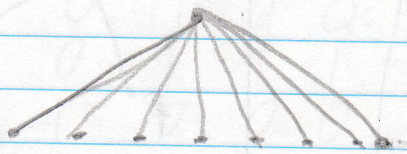
K_7



$K_{1,8}$



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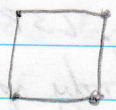
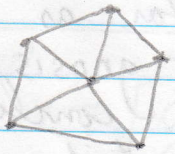
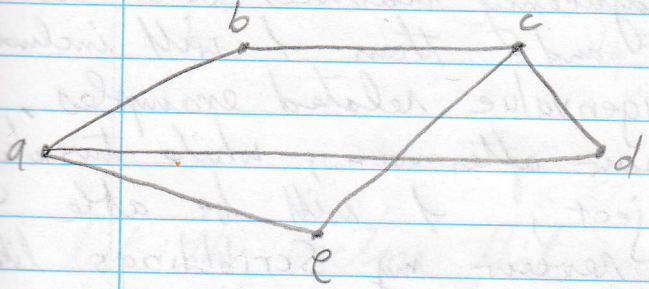
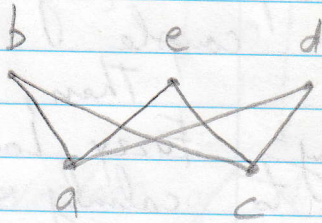


W_5



Make bipartite





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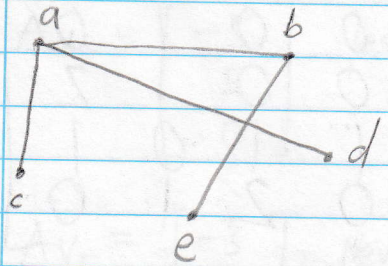
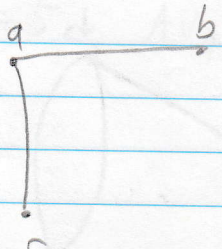
"is contained in"

$G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$



G_1

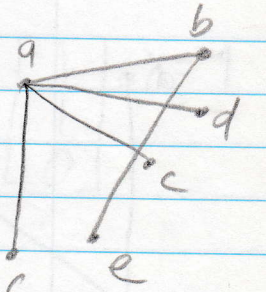


G_2

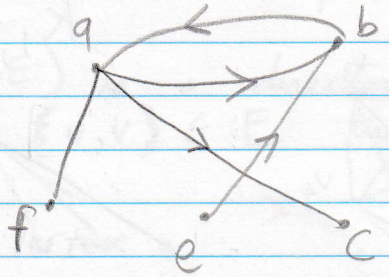
total of 6 vertices

a & b get counted once

$G_1 \cup G_2$



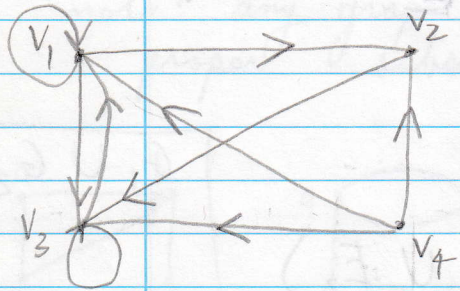
Note that the graph below is still "simple" even though it is directed, the reason being that there are no more than 1 edge per direction between each set $\{u, v\}$ and $\{v, u\}$



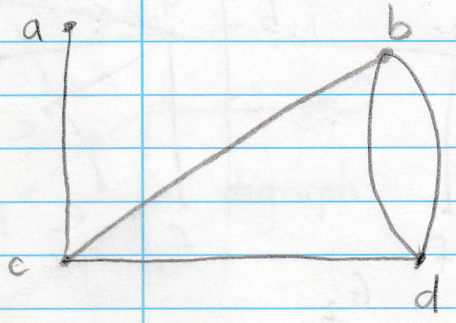
Also, in the union of two simple digraphs, there is no need to count $a \to b$ twice. This is by definition of UNION.

As far as Matrices go, I began to transcribe some gnosis herein. See p. 238 to 242. In conclusion of this main body of my documented preparation for the second and final Discrete Mathematics exam, I will show a few examples of adjacency matrices used to represent graphs; and then I will include a couple eigenvalue related examples.

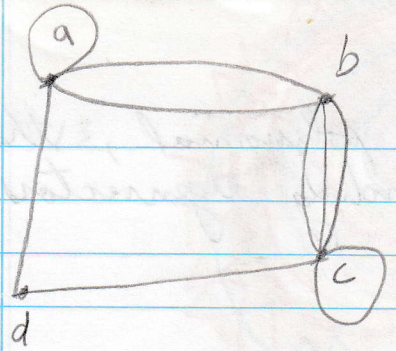
Then, tomorrow afternoon, while tweaking my Fuzzy Logic Project, I will be able to calmly review my Scribblings like a monk or a witch studying some secret formulas.



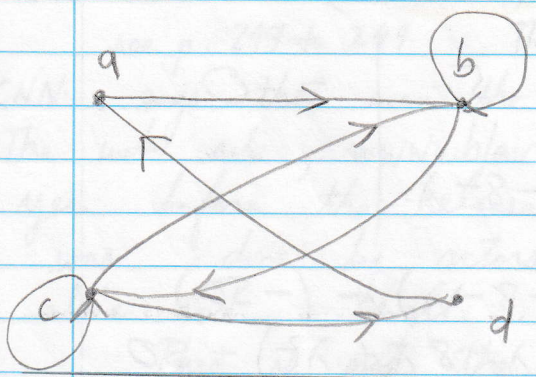
	v_1	v_2	v_3	v_4
v_1	1	1	1	0
v_2	0	0	1	0
v_3	1	0	1	0
v_4	1	1	1	0



	a	b	c	d
a	0	0	1	0
b	0	0	1	2
c	1	1	0	1
d	0	2	1	0



	a	b	c	d
a	1	2	0	1
b	2	0	3	0
c	0	3	1	1
d	1	0	1	0



	a	b	c	d
a	0	1	0	1
b	0	1	1	0
c	0	1	1	1
d	1	0	0	0

Consider matrix and vertices

$$A = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$v_4 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad v_5 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

list the vectors that are eigenvectors of A. give the eigenvalue in each case.

EXPERIMENT TI-85 yields eigVl $A = \{5, 1, 5\}$

eigVc $A = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Now, using pencil: $Ax = \lambda x$

$Av_1 = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$ Never ; $Av_2 = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$ yes, when $\lambda = 5$

$Av_3 = \begin{bmatrix} -5 \\ 5 \\ 0 \end{bmatrix}$ yes, when $\lambda = 5$; $Av_4 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$; $\lambda = 1$

$Av_5 = \begin{bmatrix} -3 \\ 2 \\ 5 \end{bmatrix}$ Never

Lastly, find the characteristic polynomial, the real eigenvalues, and the corresponding eigenvectors of the given matrix.

$$A = \begin{bmatrix} 7 & 5 \\ -10 & -8 \end{bmatrix} \qquad Ax = \lambda x \qquad \det(A - \lambda I) = 0$$

$$\det(A - \lambda I) = \begin{vmatrix} 7-\lambda & 5 \\ -10 & -8-\lambda \end{vmatrix} = 0$$

$$\begin{aligned} \det(A - \lambda I) &= (7-\lambda)(-8-\lambda) - (-50) \\ &= (-56 - 7\lambda + 8\lambda + \lambda^2) + 50 \\ &= \lambda^2 + \lambda - 6 = 0 \end{aligned}$$

CHARACTERISTIC POLYNOMIAL

$$(\lambda + 3)(\lambda - 2) = 0 \qquad \left.\begin{aligned} \lambda_1 &= -3 \\ \lambda_2 &= 2 \end{aligned}\right\} \text{EIGENVALUES}$$

Eigenvectors associated with $\lambda_1 = -3$

$$Ax = -3x \qquad \begin{aligned} 7x_1 + 5x_2 &= -3x_1 \\ -10x_1 - 8x_2 &= -3x_2 \end{aligned}$$

$$\left.\begin{aligned} 10x_1 + 5x_2 &= 0 \\ -10x_1 - 5x_2 &= 0 \end{aligned}\right\} \text{choose } \begin{aligned} x_1 &= 1 \\ x_2 &= -2 \end{aligned}$$

eigenvectors: $\begin{bmatrix} 1 \\ -2 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

check $\begin{bmatrix} 7 & 5 \\ -10 & -8 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} \stackrel{?}{=} -3 \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$$\begin{bmatrix} -3 \\ 6 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \end{bmatrix} \surd$$

$Ax = 2x \qquad \begin{aligned} 7x_1 + 5x_2 &= 2x_1 \\ -10x_1 - 8x_2 &= 2x_2 \end{aligned} \quad \left.\begin{aligned} 5x_1 + 5x_2 &= 0 \\ -10x_1 - 10x_2 &= 0 \end{aligned}\right\}$

$x_1 = 1$
$x_2 = -1$ eigenvector associated with $\lambda = 2$ is $x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

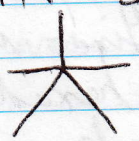
Too much time spent alone... Just as well I will be working as a programmer. Maybe I will earn enough money to go camping every now and then. I am bored but boredom is better than pain, and Schopenhauer teaches that life fluctuates between the two.

I feel there is nothing in this town for me. I have been in Freehold since I was 4. 32 - 4 = 28 years - It has changed drastically.

Now, after accidentally escaping from my state of slavery with the local park (state job) I find myself engrossed in LEARNING. Education has become the central theme of my existence; and yet I feel anxious to get a job as a 'real-time' computer programming geek.

Yes! but I would love to study about Computer Architecture and compilers. Maybe I could get a scholarship to go to Rutgers or something.

Fuck that. I want a job.



I guess some people are just MISERABLE. Maybe I am into mental stimulation because it allows my mind to get into abstract concepts. Sometimes I feel like an infant, unhappy "just because" and I don't want a solution to my dilemma. dilemma?

I am one of the masses, a consumer trying to upgrade his lot in life. My sleeping pattern is abnormal. I take in too much caffeine and nicotine.

I don't think I can stand another 3 years in limbo going for a Bachelors degree. I wouldn't mind dying in my sleep. And yet I look forward to more learning.

00:40 hrs And so I "pop" 3 sleeping ~~pills~~ pills, an Exedrin PM, and 300 mg of Lithium. Lithium — my ticket to an associates degree in Computer Science. I couldn't have paid for my education without NJ DVR. Being diagnosed as a manic-depressive really got me in the door; and yet it was my own discipline that enabled me to pull a 3.4 up to a 3.8 in one year. (In one year I did 4 semesters!)

So, in the midst of my accomplishments, I guess I am actually blind to them. I can't fully appreciate whats taking place.

Will I ever be happy? How many people are actually happy? And what about the traffic and conjestion? The OVERpopulation? It's hard to fathom that every single one of these mother fuckers is as concerned with its own agenda as I am about mine.

I have tried. I tried working for the State Park Service. Where did it get me? Smoking crack in a historic house, on the verge of suicide. Sick member. There is no doubt in anyone's mind that I am better off out of that house, away from the daily insanity of NBSP — with it's god damned Region Office and Trenton Office. What was I? — some kind of idiot sold into white slavery because of my 1987 arrest? And I was to look upon my situation as one who had won the lottery? If I was so blessed, why was I suicidal?

And what about the enormous contradiction that my verbal and mathematical skills are evidence of? The contradiction being that I sat on a tractor driving in circles getting baked by the sun while my intellect rebelled. I sabotaged my cush job with the State govt.

Now that same intellect that rebelled against being enslaved (in a civilized manner of course) is being stimulated by learning; and yet there is something missing. I am in limbo. Without an occupation, I have only my studies to give me dignity. That job at MBSP gave me dignity. I drove that new truck. I lived in that huge old historic house. Sherry really gave me some dignity; but that was long ago... too long ago to be figuring out where it went bad now, four years later at 1 AM.

I know drugs helped to destroy me, alcohol included; but there is a psychological/psychiatric/mental/emotional aspect that supercedes, and is actually at the root of, my drug abuse 1994-1997.

Don't lie. You did not want to marry Sherry Nevulis. You didn't even like her. You just liked the SECURITY of having her presence AROUND. The freefer gave you back your mind. The calculus formulas began to take on a religious flavor, as though you were a wizard of old, one thousand years behind, trying to remember who she was.

Don't sell yourself short. You have been through a lot, and it has made you stronger.

WAIT. What is it that writes as though it were speaking to me, even though it is my hand that moves the pen, my brain that "catches" the thoughts?

* What is this "my" crap? Whose hands? Whose brain? Who are "you" anyway? You have only an inkling!

214 23:40 hrs ◎◎ I made it into the ocean via spontaneity. I got to the school at 18:02 and there was a note on the door: MA226 class cancelled tonight. And so I went to the library to do some research on fuzzy Logic — I am trying to get a better grasp on DEFUZZIFICATION before I have to do the presentation. I also want to write (in my paper) a little about the general complexity involved in massaging fuzzy "analog" input into crisp "discrete" output. I will use the downloaded information ... over the weekend.

Note: I saw the Indian (Asian) girl from my math class in the library. She had the same idea as I did I guess (about getting some research done.) She smiles at me while she speaks. She has a very warm smile, and she is very shy. I like her, but that is as far as it will go. I have learned from my experience with Francis Soto in 1995 (see end of α, beginning of β). Cultural diss —

Anyway. I decided to seize the day/night. I picked up my nephew at 8:30, that is 20:30 hrs, and we drove down to Spring Lake to be baptized by the Ancient Ocean ♄; under the stars, in the warm August waters ... I am alive.

My insurance installment (660) is paid ✓ My rent and utilities are paid ✓ My pet projects are completed to be turned in tomorrow night ✓ I am prepared for my final exams ✓ I have one more weekend to prepare my Fuzzy Logic Project speech.

I am working with my nephew again tomorrow — another $80.00. I will be able to buy a new tire for Socrates on Wednesday! I don't know about the rear shocks, but at least a tire for now. Now, perhaps I will jot down some notes — some proofs to go over while cleaning tomorrow

23:10 hrs Oh, to live inside the skin of a sensitive, moody bundle of nerves! And it is I who has to endure my own experience of the human condition!

Note: After August 11th, when the semester ends, visit (1) Greg Gilroy (2) Jason Iverson (3) Eric Bont

All day I was moody, repeating out loud, "The earth is a ball of sex and death, an unmitigated disaster, a cosmic trap ... we have been FUCKED INTO this dilemma."

See the situation honestly. Why must so many writers play the role of the decent and clear-headed voice of reason? What about ones honest inner experiences - the emotions and thoughts that are incapable of lying?

I really felt like an outsider today. I have this hostile defensiveness of being of European decent. I feel as though many of the local people that are NON Caucasian share a common resentment of "white people" — and yet I don't care for most people, white or otherwise. I am terribly aware of twisted vibrations resonating in my veins and sinews. I sense that I am viewed as "somewhat of an outcast".

Are these imagined or real? Even if my writings were to be read as pseudo-literature one day after my bones lie stinking in the earth, this would have no effect on my LIVING CONDITION right here, right now.

I look for some kind of signal in the eyes of people I encounter in the stores, but all I see is silent rejection ... or perhaps even mockery, ridicule, severe judgement. My sanity is questioned. I am not a happy man. How long does it take to grow this beard?

I look on the shelves of the Barnes and Noble for an existentialist-type book on the human condition, but nothing grabs me. I return to my abode... I very much self-reflective, very much the philosopher/psychologist. I am fascinated and disgusted with the vision of my state of being. At once I am terrified, hostile, and yet still tender and enlightened. So much hostility in my cold heart — but why am I so concerned then? I have peered deep into the abyss, and I have seen that the "unseen forces" do favor me. I have been blessed with painful learning experiences that give me a more honest perception of the condition "I"/"we" are experiencing.

Even in the county jail, when a decent meal is served, there is relief. There is freedom when we accept that which is outside of our control.

I believe it is outside of my control that I lust after beautiful women. I believe it is outside of my control that I do not have any luck with attaining the fruit of my desires. I do not control my blasphemous hostility towards ~~trust~~ Judaism and Christianity — and Islaam too god dammit. Jehova 1 Space God = Allah the Ignoramus Demiurge. These are only words, and yet they are words free from the scrutiny ~~strutiny~~ of academicians, scholars, and critics. These are the words of the only LORD of thy world — the eater of food, that which EXPERIENCES mood swings, social confusion, and the general suffering of daily existence.

I am tired, and this Great Tiredness is good. Shall I surrender to my genuine depth?

220 23:00 hrs I have tried everything from setting up my linux machine as a workstation to Custom. startx is not loading at \root#. I am thouroughly disappointed. I was to drop off the LINUX cd at 9PM, and here I am — still hacking away at it. I wonder why my nephew hasn't called. He must be furious by now. I figure, "He got me into this jam. He will have to wait."

If this doesn't work, I will return the cd before midnight — along with the book I just purchased at Barnes and Noble, "Red Hot Linux 6".

7 minutes to go ... then I set up the mouse and monitor ... phone rings (nephew)

23:45 Bad news. Complicated situation. My sister is angry that my nephew has not completed his Algebra II assignments. He has been taking Algebra II since last September I think! He works alot for his parents ... My sister thinks he spends too much time on the computer. She yanked the cord from the ~~monitor~~ keyboard to the cpu — as she pulled the cord, his cpu fell to the floor. Now it only blinks on the terminal. She ~~through~~ threw water in his face — the water went all over his algebra work. If he can just wait until August 17th, I will help him with his algebra, AND we can take a look at his machine even if it means hooking it up over here in my "lab".

I guess I won't be returning his LINUX cd after all tonight. The real heart freaking aspect of this is that I do believe his constant computing and god forbidden net surfing does impede his algebra studies as does his working for his parents! I am afraid he will not fare well with Physics and Pre-Calc

I fear for his life. I would be devastated were he to take his own life. I will help him finish his Algebra II work... I will help him with his Physics in September. I want to see him go off to college next year, away from those fucked up Christians.

Did I mention my sister may have a good point? I am actually very aware of my nephews "lack of commitment" to his studies. He is more interested in hacking. The situation is complex, fuzzy — not black and white. The dynamics of the situation are heartbreaking.

1999 221 MONDAY 09 AUGUST 00:30 hrs

One final attempt to load Linux before I try the QRestore Factory installation again. root@lucifer666 startx unable to connect to X server.

One more try with Linux.

① delete all partitions first.
- how? Fdisk will not allow it.
- run fips from cd:\dosutils
- fips splits 'em. It does not delete them.
- delete all partitions with LINUX CD.

Fuck it. Try Linux again!

I can't concentrate on math. I am too caught up with getting an OS to work on my machine — besides UNIX. I repeat, I would be content just to get my Factory defaults back!

If Linux's startx does not take this time, if I get no X windows, I will load the LINUX cd again, remove the partitions, and then try Factory restore again. Then, if it fails, I will just go to sleep.

01:30 formatting hard drive manually. If this does not work, I am quitting for the night.

02:30 LOCKS UP at step 4 (p199)

Logbook #54 (20 JAN... 27 FEB 1998) contains, very near the beginning (20 JAN) my first call to NJ Employment services when I was directed to NJ DVR on 60 Taylor Ave [775 1497]. Little did I know that Jane Murray and company would be giving me a chance to continue my education full time. DVR has made my life worth living. The knowledge I am gaining is priceless. As I was working on a collection of excerpts back while writing L54, it ties things together. Perhaps I will pick away at some excerpts, before leaving logbooks #'s 1 to 52 behind in Mom's basement.

Logbook #55 contains the reflections during a part of my life when I was initially "tested" by DVR, when they realized my potential, as well as when I first registered for classes at Brookdale

I wish to include here an excerpt from L55 which will make sense of my decision to leave notebooks 53, 54, and 55 behind with the rest:

From April 18th, 1998

"Just as the 12 legal pads written in while I was held captive in the Monmouth County Jail served as a verbal bridge between July 13th 1997 and November 20th 1997, so too do the last four notebooks, including this new one, serve simultaneously as a conclusion to my "State Park Years" and an introduction to "a new era of my life". With this notebook <LOGBOOK #55>, I close these chapters of my life. I am surrounded by people who tell me that leaving the park service is a great blessing, that this will free me so that I may develop my intellect, leaving the tractor behind me.

There is a reason why I wrote in 3 LARGE RECORD BOOKS!

It's an ending. It is all part and parcel an ending, with the beginning being the very ending process of one phase of my life. "Portraits" – LOGBOOK #56 represents the beginning of my quest for degrees... I will bring Logbook #'s 56, 57, 58, 59.

// primes
// mw hentrich

```
#include <iostream.h>
void main()
{ long n = 4;
  long factor;
  int i = 1;
  int k;
  int p[1000];
  p[0] = 2;
  p[1] = 3;
  char ch == 'y';
  int prime;
  int count = 2;

  cout << endl << endl << "2 = 2" = prime #1,
  cout << endl << p[1] << " = prime #2";
  while (ch == 'y') {
      prime = 1;      // set "prime flag" to ON
      k = 1;          // p[1] = 3, the first odd prime
      count++;
      cout << endl;
      cout << n << " = ";
      factor = n;

[1]   while (factor > 1) {
[.]       if (factor % 2 == 0) {   // even?
              prime = 0;   // prime flag OFF
              while (factor % 2 == 0) { // 2 | factor
                  factor /= 2;   // factor ← factor / 2
                  cout << "2 ";
              }
              while ((factor > 1) && (factor % 2 == 1)) {
                  while (factor % p[k] == 0) {
                      cout << p[k] << " ";
                      factor /= p[k];
                  }
                  if ((factor > 1) && (n/p[k] != 0)) k++;
                  else { factor /= p[k]; }
[-]       } }
```

(logic error)
check out "25
45, 77, etc

This is not the place to write code. I made minor changes. I will have to organize hard copies of my programs in a code (log book) folder.

```
                    cout << p[k] << " ";
                    factor /= p[k];
                    prime = 0;          // p[k] | factor
```

```
else    // if not even
    if (factor % 2 == 1) {
        while (factor > 1) {
            if (factor % p[k] == 0) {
                while (factor p[k] == 0) {
                    cout << p[k] << " ";
                    factor /= p[k];
                    prime = 0;      // p[k] | factor
                }
                if (factor > 1) k++;
            }
            else
                if ((n/p[k] <= p[i])
                    && (prime == 1)) {
                    i++;
                    p[i] = n;
                    cout << p[i] << " ";
                    factor /= p[i];
                }
            else
                if ((factor % p[k] != 0)
                    && (prime == 0)) {
                    cout << factor;
                    factor /= factor;
                }
        } // end while
    } // end if (factor % 2 == 1)
} // end second while → factor > 1 ?
n++;
if (i >= 999) break;
if (count == 20) { count = 0; cout << "continue?"; cin >> ch; }
} // end while ch == y
```

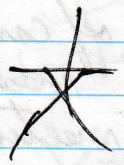
end main

NO SIDEBURNS

(AND 1 cm. ABOVE THE EARS)

~~255~~ 1999 256 MO 13 SEPTEMBER

10:00 hrs reading Mark Twain's "Letters From the Earth" — I have 2 copies... gave one to Jmm. Powerful stuff. Twain - I have never seen this side of Mark Twain put forth with such sarcasm and wit. I am inspired by Satan (Mr. Samual Clemens himself!)



256 13:00 hrs HOLY SHIT! Holy fucking shit! I called Kathy Lambertson after my coffee at around 12:30! Bottom line: DVR will not only sponsor me for tuition at Rutgers — they will grant me up to $1300 per semester, for room and board (even if I have to lease a room off campus), as well as $800 per semester towards a meal plan (so I can eat on campus)! I would get a part time job to pay for my car insurance, my fines, and whatever else I might need... !!!

Holy shit.

I called CBA about my transcript. Answering machine. I will begin typing a letter to Rutgers admissions. I will mention that, because of my age, I would be better off living off campus; and that CBT transcript is on its way!

(So, I sense some cold realities biting at me. I feel that while Chuck Sang and Steve Dubois are genuinely happy for me — José Sanchez (and possibly even Ed Henderson) may feel some resentment towards me — like I am getting a free ride or something.

Jose's comment: "So what you're telling me is that, even though I can't afford to send my kids to college, I have to pay for your to go to college? That's fucked up."

- Yes, Jose. You loved me when I was down and out, when I was a jailbird; but now that the tides are turning and I may be rising in social stature, now you want to curse me. How shallow you are. How false your outward display of sympathy! You must hate me if you want to deny me this opportunity. You know what? I would rather be moving towards a goal such as a Bachelors degree in Computer Science AND be cursed, resented, and bad mouthed than to be sitting in jail with sympathy from all. Everyone loves the fool. genius.

Hardly any likes the success story. I want to understand the base nature of human psychology. Only then will I be able to (would) comprehend how my good fortune will invite scorn, while my hard luck invites compassion.

Make up your fucking minds. If you don't wish me well, they don't; but if you do wish me well, they rejoice when those wishes are granted!) Fuck.

Now... Tom Sandle and Paul Sedor? They have a new respect for me.

And that was ch 23 — the stuff I kind of understand.
Here is what the Universe shall introduce me to this week:

Electric flux is represented by the number of electric field lines that penetrate a surface. If the electric field is uniform and makes an angle θ with the normal to the surface, the electric flux through the surface is:

$$\Phi = EA\cos\theta$$

In general, the electric flux through a surface is

$$\Phi = \int_{\text{surface}} E \cdot dA$$

Gauss's Law says that the net electric flux, Φ_c, through any closed gaussian surface is equal to the net charge inside the surface divided by ϵ_0:

$$\Phi_c = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

Charge Distribution	Electric Field	Location
insulating sphere of radius R, uniform charge density, and total charge Q	$k_e \frac{Q}{r^2}$	$r > R$
	$k_e \frac{Q}{R^3} r$	$r < R$
thin spherical shell of radius R and total charge Q	$k_e \frac{Q}{r^2}$	$r > R$
	0	$r < R$
Line charge of infinite length and charge per unit length λ	$2k_e \frac{\lambda}{r}$	outside the line charge
Non-conducting, infinite, charged plane with charge per unit area σ	$\frac{\sigma}{2\epsilon_0}$	everywhere outside the plane
conductor of surface charge per unit area σ	$\frac{\sigma}{\epsilon_0}$	just outside the conductor
	0	inside the conductor

Note: If my notebook for Operating Systems class is officially "Brainwaves_2", what is my Physics II notebook - or my Physics II Work Sketch Diary? BRAINWAVES_PHYS122NOTES

01:30 hrs We experience the miserable pressure of the will first hand, on a purely subjective level. We don't need any psychologist ~~as~~ nor priest to tell us "How to refuse to be miserable about anything, yes anything!" Our inner experience of the will does not know the art of deception.

Could it be that philosophy, true philosophy, such as Schopenhauer's, is higher than natural science (physics) and mathematics? Are not math and science merely tools used by the will to secure its desires, to increase or strengthen its CONTROL, its illusion of control over the world outside its skin? Is there really any way to know this world outside the skin, when all we experience is 100% inside our skin, in perception and sensation?

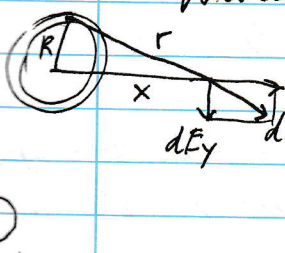
I go too far?

I was in bed with my eyes closed, when I went through a series of flash backs... some recent, some distant. I felt what can only be described as pain, pain in my "heart", emotional stress, distress. Images of my last weeks ~~as~~ employed by the Park Service, my psychotic episode, the arrest of my organism, my release, my dismisal from public service, my grant to go to college (with deep sweet gratitude, almost painful - like love); I also felt the presence of my father, my deceased grandfather (Hentrich), my nephew...

I fear that my father thinks I do not look up to him, that I think I am "smarter"... this brings me pain because I feel my father's pain. ~~for~~ I feel pain for my nephew because I know he is struggling with a sense of identity - and nowhere does he receive any affirmation of his declarations of intelligence. He is thinking!

He is aware of his own awareness. So I could not just keep my eyes closed. I felt compelled to turn on the electric lights, pick up a pen, and translate emotive stirrings into verbalizations. The city also disturbed me. Millions of "souls" suffer daily. Each becomes tough, but the reality of human MISERY cannot be denied.

Within me the PHILOSOPHER still screams.



is all well and true and beautiful, but there is no rational way to compute the meaning of our very being, our non being even.

$dE_x = dE \cos\theta$ This is the "x component"

Here $\cos\theta = \frac{x}{r}$ and $r = \sqrt{x^2 + R^2}$

These truths I am becoming familiar with, and I recite them as a priest reciting his creed; these truths are abstract. Yet there are truths that escape abstract symbols and verbalizations.

Perhaps this is why science calms me.

To continue with the spherical shell and its E — its electric field: $dE = k_e \frac{dq\ x}{r^3}$

where k_e is "Coulomb's Constant" $\approx 8.99 \times 10^9 \frac{N \cdot m^2}{c^2}$.

We write $E = \int dE$. I write this slowly. It brings me a sense of peace. I experience the beauty ... so, if $dE = k_e \frac{dq\ x}{r^3}$, then

$$E = \int dE = \int k_e \frac{dq}{r^2} = \frac{k_e\ x}{r^3} \int dq = \frac{k_e\ x\ q}{r^3}$$

Note that $r = \sqrt{x^2 + R^2} = (x^2 + R^2)^{1/2}$,

hence, $$E = \frac{k_e\ x\ q}{((x^2+R^2)^{1/2})^3} = \frac{k_e\ x\ q}{(x^2+R^2)^3}$$

Would Guass' Law work?

$E = \frac{q}{\epsilon_0}$???

Oh, I admit that I am soaking up this high knowledge like a sponge; but my love for learning cannot cancel out the little constants of pain that are inherent in breathing, eating, striving to survive, chasing our desires.

I see the doubt and confusion in my nephew's eyes. He is experiencing the pain of consciousness, the very burden of existence itself — the principium individuationis. I don't know how to help, as I myself in caught in the same Web, the same metaphysical prison.

He does not consume alcoholic beverages, nor does he ~~inn~~ injest mind altering substances like cannabas, opium, nor cocaine. I wonder if he realizes how much worse he could feel. The potential for pain is mind boggling. It is all relative. Each individual's pain is the mythical crucifixion.

And am I surprised that the miserable pressure of the Will is still a KNOT in my guts? Why would it leave me? Do I think that education or improved social position would relieve me of the very NATURE OF THE HUMAN CONDITION ITSELF?

I do admit that I am much more enthusiastic about continuing my education in SCIENCE and MATHEMATICS than I was with being bullied by blockheads, windbags, and even total morons when I was a government janitor, defender of public toilets, Mower of the Grass (as well as loyal Smoker of the Grass).

In the morn', I hope to review some Physics problems before I go to the campus; but tonight, even as it is after 2AM, I am drawn to Philosophy — Schopenhauer's philosophy. I am drawn due to the growing awareness that the fact that We experience the Will-to-live (and the

world itself, the thing in itself) directly makes each individual an EXPERT on reality — whatever it is reality may be. Reality turns out to be a subjective — a purely subjective-experience.

No one can measure another's subjective pain.

I think that, perhaps, keeping track of my thoughts on a daily basis, may help that which eats food get a glimpse of what we call THE UNCONSCIOUS.

In fact, the greatest value of a diary is its Revelations — its spontaneous exposure of unconscious elements.

Perhaps I awoke this evening to write a short phrase concerning my nephew only so that THE UNCONSCIOUS (call it god, call it satan, call it universal mind, whatever) could communicate with ITSELF — itself being in the vessel which happens to be my nephews head. But if he is reading these very words in the distant future — and the writer of these scribblings is "dead", then these words — these ideas are in his head.

Now, if these ideas are in "a" head, then these words are in "a" "my head". These ideas are in MY HEAD, "MY" being THE UNCONSCIOUS. Aha, so there is a bit of "metaphysical truth" even in Christianity... Let's not go there.

But it doesn't have to be my nephew's brain. It is MY HEAD regardless.

IDEAS cannot escape their physical nature. IDEAS are electro-chemical. They reside in brains. Without a brain, there is not a Thinking Being (soul). Do the ideas themselves transcend the principium individuationis?

||

These are not wasted days, these days without work. I use my time wisely. SLACK grants me time to be myself, and in the raw, I am a studious thinker, a philosopher who is mainly interested in computing — and therefor interested in mathematics and digital computers. Is my lack of a full time job evidence of my LAZINESS? What the fuck is LAZINESS? Can a man who stays awake until 6AM working on a computer program be considered lazy? If my energies do not produce money, then my energies seem to lack value. Our culture values money. Money leads to securing our life and limb. These are NOT wasted days! These are not wasted years. The time (the SLACK) I am stealing now is being put to tremendous use! The CRAFT I am learning will provide me with financial security eventually. Yes, this craft will become my income.

For now, there is minimal income, and it surely does not come from my computing. The bottom line is that poverty sucks. There are surely several valid responses to poverty, but the main response seems to be, to work for money; even more than one job. This is the standard response.

Another alternate response is to wade deeper into poverty by becoming a STUDENT, yes, a college student. This will make matters worse for however long it takes to become a highly skilled professional. The result of the latter route is to claim a place in the workforce as a skilled member. The result of the former is to bank money, but never really RISE. Besides, being a student has rewards on a daily basis (intellectual stimulation), not just the final rewards.

$$\Delta U = -q_0 \int_A^B E \cdot ds$$

The integration is performed along the path by which q_0 moves from A to B, and the integral is called either a path integral or a line integral. The potential energy per unit charge, U/q_0, is independent of the value of q_0 and has a unique value at every point in the electric field. The quantity U/q_0 is called the electric potential, V.

$$V = U/q_0$$

The potential difference, $\Delta V = V_B - V_A$ is defined as the change in potential energy divided by the test charge q_0:

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B E \cdot ds$$

Potential difference, ΔV, should not be confused with difference in potential energy, ΔU.

$$\Delta U = -q_0 \int_A^B E \cdot ds \; ; \quad \Delta V = \Delta U / q_0 = -\int_A^B E \cdot ds$$

The unit of potential is [energy per unit charge] is joules per coulomb, defined to be equal to a unit called the volt (V). $1\ V = 1\ J/C$.

1 J of work must be done to take a 1-C charge through a potential difference (ΔV) of 1 V.

The SI unit of Electric field E (N/C) can also be written as volts per meter: $\frac{N}{C} = \frac{V}{m}$

notes on units:

electric charge: coulomb C → A·s (amperes * seconds)

force: newton N → kg·m/s²

electric field E → N/C → $\frac{kg\ m}{A\ s^3}$ → V/m

work: joule J → kg m²/s² → N·m

electric potential: volts V → kg m²/A·s³

It is best to remember these in terms of their fundamental SI units.

hence E: $\frac{N}{C} = \frac{kg\ m}{A \cdot s^3} = \frac{V}{m} = \frac{kg\ m^2}{A\ s^3\ m} = \frac{kg\ m^2}{A \cdot s^3} = \frac{N}{C} = \frac{V}{m}$

I want to make sure I understand the DIFFERENCE between potential DIFFERENCE and DIFFERENCE in potential energy.

change in potential energy ΔU

potential difference $\Delta V = \frac{\Delta U}{q_0}$.

The key is in ds (infinitesimal displacement).

potential energy is a scalar function of position.
electric field is a vector – more complex than potential.

FACT: the ~~measured~~ difference in electric potential between 2 points is in fact the measured voltage between any two points in an electric circuit.

electric potential is measured in volts. It is voltage.

$V \rightarrow \frac{Kg\ m^2}{A \cdot s^3} \rightarrow \frac{J}{C} \rightarrow \frac{N \cdot m}{C} \rightarrow \frac{WORK}{charge}$

change in potential energy is ΔU where U is the work done by the electric field: $F \cdot ds$ $(N \cdot m)$ $(N \cdot m) \rightarrow J$; in fundamental terms, work is measured in units $Kg\ m^2/s^2$. This is the nature of ΔU → change in potential energy, where potential energy is "work".

Divide ΔU by q_0 and we have ~~energy~~ potential – that is, electric potential V.

The units of measurement are the glue that can really bind these concepts together for the student.

$q_0 E = F = ma$; $F \cdot ds = q_0 E \cdot ds$; $dU = -q_0 E\, ds$

in fundamental units: $(A \cdot s)\left(\frac{Kg\ m}{A \cdot s^3}\right) \rightarrow \frac{Kg\ m}{s^2}$

$\left(\frac{Kg\ m}{s^2}\right)(m) \rightarrow \frac{Kg\ m^2}{s^2} \rightarrow (A \cdot s)\left(\frac{Kg\ m}{A \cdot s^3}\right)(m)$

$V = \frac{U}{q_0}$

$\frac{Kg\ m^2}{s^2} \cdot \frac{1}{A \cdot s}$

Here the student is faced with a decision to be made: does he/she memorize the formula for C and ΔV in this situation or does he/she do the calculus from scratch each time?

Some things to remember (and some cool calculus "too").

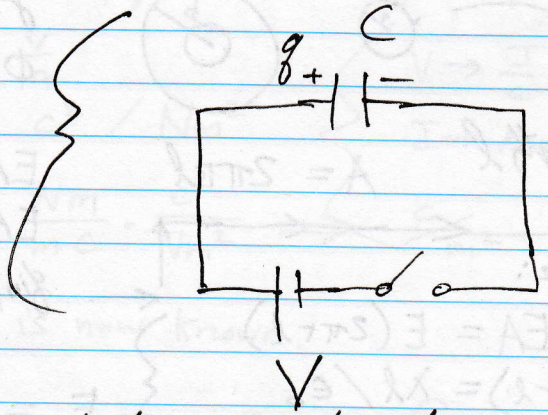
If parallel, potential V is same $V_1 = V_2 = V$
and $C_{eq} = C_1 + C_2$

If series, charge Q is same $Q_1 = Q_2 = Q$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Concepts: time increases to $t + dt$ (little bit of time)
charge increases to $q + dq$ (little charge)
is it $t + dt$? $q + dq$?

$V = q/C$



How much work needed in order to make $q + dq$?

"WORK" $= qV$ $\therefore$ $dW = dq\,V = \frac{q}{C}\,dq$

$$\int dW = \int V\,dq = \int_0^Q \frac{q}{C}\,dq \qquad \left\{ \int q\,dq = \frac{q^2}{2} \right.$$

$$= \frac{q^2}{2C}\Big|_0^Q = \frac{Q^2}{2C}$$

AWESOME CALCULUS!

$Q = CV$

$$W = U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}\frac{C^2V^2}{C}$$

$$= \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{Q}{V}\right)V^2 = \frac{1}{2}QV$$

Hence all these are equal.

$$W = U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

$Q = CV$ ... $C = Q/V$ for parallel plate $C = \frac{\epsilon_0 A}{d}$

interesting to note (see p. 476)

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0} \qquad V = Ed = \frac{Qd}{A\epsilon_0}$$

$$C = Q/V = Q/Qd/A\epsilon_0 = \boxed{\frac{A\epsilon_0}{d}} \quad !$$

$V \rightarrow$ voltage
$\mathcal{V} \rightarrow$ volume

Now, $U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{A\epsilon_0}{d}\right)V^2$

tricky part: multiply by $\frac{d}{d} = 1$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{A\epsilon_0 d}{d\,d}V^2 = \frac{1}{2}\frac{A\epsilon_0 d\,V^2}{d^2}$$

SOME ALGEBRAIC MANIPULATION: $V = Ed$; $E = V/d$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}A\epsilon_0 d\left(\frac{V}{d}\right)^2 = \frac{1}{2}A\epsilon_0 d E^2 \rightarrow \frac{1}{2}A\epsilon_0 d E^2$$

Now, Ad implies VOLUME $\rightarrow \mathcal{V}$, as Area is m^2 and d is m, hence $\mathcal{V}$ m^3 (units).

and $U = \frac{1}{2}\epsilon_0 \mathcal{V} E^2$

↑ how much energy the electric field carries.

introducing u: $\frac{\text{energy}}{\text{volume}} \Longrightarrow$ energy density u

$$\frac{U}{\mathcal{V}} \rightarrow \frac{J}{m^3} \rightarrow u = \frac{1}{2}\epsilon_0 E^2$$

STOP! Do this in a scrap pad! These are the last precious pages of LSB, not some BRAINWAVES notebook or scratch paper!